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PSEUDO-UMBILICAL SUBMANIFOLDS OF CODIMENSION 2

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Dedicated to Professor Hiraku Tôyama on his sixtieth birthday

The purpose of the present paper is to study the so-called pseudo-umbilical submanifolds of codimension 2 in Euclidean and Riemannian manifolds. Our main results appear in Propositions 2.3, 3.2, 3.3, 4.1, 4.2 and 4.3.

In § 1, we reformulate formulas for submanifolds of a general Riemannian manifold and, in § 2, we specialize these formulas to those for submanifolds of codimension 2 of a Euclidean or a Riemannian manifold.

We study, in § 3, pseudo-umbilical submanifolds of codimension 2 in a space of constant curvature and, in § 4, those in a Euclidean space. In the last section 5, we prove, for the completeness, some of lemmas which are used in the paper.

§ 1. Formulas for submanifolds.

As we are going to study some special kinds of submanifolds, we would like first of all to reformulate formulas for submanifolds of a Riemannian manifold for the later use. Let M^n be an n-dimensional manifold[1] differentiablly immersed as a submanifold of an m-dimensional Riemannian manifold M^m, where $n<m$, and denote by x: $M^n \to M^m$ the immersion. Denote by B: $T(M^n) \to T(M^m)$ the differential of the mapping x, i.e., $B=dx$, where $T(M^n)$ and $T(M^m)$ are the tangent bundles of M^n and M^m respectively. On putting $T(M^n, M^m)=BT(M^n)$, the set of all vectors tangent to $x(M^n)$, we see by definition that B: $T(M^n) \to T(M^n, M^m)$ is an isomorphism, since $x: M^n \to M^m$ is an immersion. The set of all vectors normal to $x(M^n)$ forms a vector bundle $N(M^n, M^m)$ over $x(M^n)$, which is the normal bundle of $x(M^n)$. The vector bundle over M^n, which is induced by x from $N(M^n, M^m)$ is denoted by $N(M^n)$ and called the *normal bundle* of M^n with respect to the immersion x. We now denote by C: $N(M^n) \to N(M^n, M^m)$ the natural isomorphism.

We now introduce the following notations: $\mathcal{T}^r_s(M^n)$ is the space of all tensor fields of type (r, s), i.e., of contravariant degree r and covariant degree s, associated with $T(M^n)$. $\mathcal{T}(M^n)=\sum_{r,s}\mathcal{T}^r_s(M^n)$ is the space of all tensor fields associated with $T(M^n)$. $\mathcal{N}^r_s(M^n)$ and $\mathcal{N}(M^n)=\sum_{r,s}\mathcal{N}^r_s(M^n)$ denote the respective spaces associated

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1) Manifolds, mappings, functions, tensor fields and any other geometric objects we discuss are assumed to be differentiable and of class C^∞. We restrict ourselves only to connected submanifolds of dimension $n \geqq 2$.