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COMPLEX SUBMANIFOLDS OF THE COMPLEX PROJECTIVE SPACE WITH SECOND FUNDAMENTAL FORM OF CONSTANT LENGTH

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1. Statement of result.

In a recent work [1] Chern, do Carmo and Kobayashi have established a pinching problem, with respect to the length of the second fundamental form, for compact minimal submanifolds of a sphere and have classified compact minimal submanifolds of a sphere whose lengths of the second fundamental form are certain constants.

In the present paper we shall give a complex analogue. Let $P_{n+p}(\boldsymbol{C})$ be the complex projective space of complex dimension $n+p$ with the Fubini-Study metric. Let M be an n-dimensional compact complex submanifold of $P_{n+p}(\boldsymbol{C})$ and let h be the second fundamental form. We denote by S the square of the length of h. Then we can see that

$$\int_M \left\{ \left(2-\frac{1}{2p}\right)S-\frac{n+2}{2}\right\} S\,dv \geqq 0,$$

where dv denotes the volume element of M. It follows that if

$$S \leqq \frac{n+2}{4-1/p} \qquad \text{everywhere on } M,$$

then either

(1) $$S=0 \quad \text{(i.e., } M \text{ is totally geodesic)}$$

or

(2) $$S=\frac{n+2}{4-1/p}.$$

The purpose of the present paper is to determine all compact complex submanifolds M of $P_{n+p}(\boldsymbol{C})$ satisfying

$$S=\frac{n+2}{4-1/p}.$$

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