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PERIODS OF DIFFERENTIALS AND RELATIVE EXTREMAL LENGTH, I

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Introduction.

Let R be an open Riemann surface and its ideal boundary be denoted by $\mathfrak{J}$. Let $\{A_j, B_j\}$ be a canonical homology basis modulo $\mathfrak{J}$ and $\{C_j\}$ be a homology basis of dividing cycles. Let Γ_h be the space of harmonic differentials on R with finite Dirichlet norm. It seems to be an important problem to decide when there exists a differential $\omega \in \Gamma_h$ which satisfies a period condition

$$\int_{A_j} \omega = a_j, \qquad \int_{B_j} \omega = b_j, \qquad \int_{C_j} \omega = c_j$$

for an arbitrarily given system of real numbers a_j, b_j, and c_j.

In the present paper we shall concern ourselves with the problem to decide the existence of such differentials in the important subspaces Γ^*_{h0}, Γ^*_{hm}, $\Gamma^*_{h0} \cap \Gamma_{hse}$ (cf. [3]) and further the more general subspaces Λ^*_{h0}, Λ^*_{hm} (see §1. 5). In the terms of relative extremal length (see §1. **10** for the definition) we shall state a perfect condition in order that there exists the differential with given periods in each of these spaces (Theorems 2. 1, 2. 2, 2. 3 and 3. 1). Further it is shown that the differential gives an extremal metric of a certain relative extremal length problem.

In the subsequent paper II, some applications of the present consequences will be shown.

The problem concerning the existence of differentials with given periods has been studied by many authors: Virtanen [16], Kusunoki [6] and Sainouchi [15], etc. They are mainly based on such the algebraic method as the orthogonalization of differentials. Our present method is quite different from these and very geometrical.

§1. Preliminaries.

1. Canonical homology basis modulo the ideal boundary. Let R be an open Riemann surface. A singular cycle is said to be a *dividing cycle*, or *homologous to* 0 *modulo the ideal boundary*, if it is homologous to a singular cycle which lies outside of any given compact set. Let $\mathfrak{H}$ and $\mathfrak{H}_{\mathfrak{J}}$ be the groups formed by the homology classes of all singular cycles and dividing singular cycles respectively.

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