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ON ANALYTIC MAPPINGS OF A CERTAIN RIEMANN SURFACE INTO ITSELF

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1. We shall be concerned with the study of analytic mappings of a Riemann surface into itself. Heins [5] showed that every non-constant analytic mapping of a Riemann surface of parabolic type with non-abelian fundamental group into itself is univalent. In the present paper we shall establish a similar result in a case of certain Riemann surfaces of hyperbolic type.

Let W be a Riemann surface of hyperbolic type, let $\mathfrak{G}_W(p, q)$ be the Green function with a pole at $q \in W$ and let π be a projection mapping of the universal covering surface W^∞ onto W. We take, as we may, W^∞ as $\{|z|<1\}$. Then $\mathfrak{G}_W(\pi(z), q)$ has the angular limit 0 a.e. on $\{|z|=1\}$. We denote by $\mathfrak{F}$ the set of all points of such kind on $\{|z|=1\}$. We say that two points z_1 and z_2 of $\mathfrak{F}$ are equivalent provided that there exists an element $T(z)$ of $\mathfrak{G}$ such that $z_2=T(z_1)$, where $\mathfrak{G}$ denotes the group of linear fractional transformations of $\{|z|<1\}$ onto itself which leave π invariant. This requirement defines an equivalence relation in $\mathfrak{F}$. We call an equivalence class of this relation an ideal boundary point of W and call the set of all points of $\mathfrak{F}$ belonging to an ideal boundary point its image. Each ideal boundary point belongs to a single ideal boundary component in the sense of Kerékjártó-Stoilow. Namely, let $e^{i\theta}$ be a point of the image of an ideal boundary point and let λ: $z=z(t)$ $(0\leqq t<1)$ be a curve in $\{|z|<1\}$ such that $\lim_{t\to 1}z(t)=e^{i\theta}$ and there exists a positive number ε satisfying

$$\left|\arg\frac{e^{i\theta}-z(t)}{e^{i\theta}}\right|<\frac{\pi}{2}-\varepsilon.$$

Then $\pi(z(t))$ tends to a single ideal boundary component α as $t\to 1$. This α is independent of a choice of $e^{i\theta}$ and λ. We denote by F the set of all ideal boundary points of W. If the image $\mathfrak{M}$ of a subset M of F is measurable on $\{|z|=1\}$, we say that M is measurable and call $\omega_M(p)=\omega_{\mathfrak{M}}(\pi^{-1}(p))$ the harmonic measure of M with respect to W, where $\omega_{\mathfrak{M}}(z)$ is the harmonic measure of $\mathfrak{M}$ with respect to $\{|z|<1\}$.

Let M be a subset of F of positive measure. According to Constantinescu-Cornea [1], we say that M is HB-indivisible if, for any bounded harmonic function $u(p)$ on W, $u(\pi(z))$ has the same angular limit a.e. on the image $\mathfrak{M}$ of M.

Let $\{\Omega_\nu\}_{\nu=1}^\infty$ be an exhaustion of W satisfying: for each ν, Ω_ν is relatively com-

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