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A REMARK ON A TURNING POINT PROBLEM

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§ 1. Introduction.

As a turning point problem, there are two questions: 1) is there any non-singular transformation which reduces a given differential equation to a simpler form and then what types of typical equations arise by this transformation, 2) can we construct asymptotic expansions of a solution in the full neighborhood of the turning point. For the first problem, there are many references, for example Sibuya [5], Wasow [6], and Hanson [2]. For the second problem, the author developed a matching method to a considerably general type of equations [3]. To apply the matching method, we must calculate the inner solution, outer solution and connection matrix between them. In general the inner domain where the asymptotic expansion of the inner solution exists depends on a parameter ε and shrinks when ε tends to zero to one point that is a turning point itself. But in some cases, this inner domain does not shring to one point, a finite number of the inner domains cover a full neighborhood of the turning point, and then it is unnecessary to construct the outer solution if we consider only a sufficiently small neighborhood of the turning point. In this note we give a sufficient condition so as to give this case for the two examples of equations.

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§ 2. Example 1.

Let be given a differential equation of the form

$$\varepsilon^h \frac{dy}{dx} = A(x, \varepsilon) y, \tag{2. 1}$$

where ε is a small complex parameter, y is an n-dim vector, x is a complex independent variable, h is a positive integer, and $A(x, \varepsilon)$ is an n-by-n matrix such that

$$A(x, \varepsilon) = \begin{bmatrix} 0 & 1 & \cdots & & 0 \\ & & \ddots & & \\ & 0 & & \ddots & 1 \\ p_n(x, \varepsilon), & p_{n-1}(x, \varepsilon), & \cdots, & p_2(x, \varepsilon), & 0 \end{bmatrix}. \tag{2. 2}$$

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