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ON A SYSTEM OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS WITH A TURNING POINT

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§ 1. Introduction.

1° The system of differential equations to be discussed here is

$$\varepsilon^{\sigma}\frac{dY}{dx}=A(x,\varepsilon)Y, \tag{1. 1}$$

in which σ is a positive integer, ε is a small complex parameter and $A(x,\varepsilon)$ is an n-by-n matrix function holomorphic in both variables in the domain $\mathfrak{D}$ defined by the inequalities

$$\mathfrak{D}:\quad |x|\leqq x_0,\qquad 0<|\varepsilon|\leqq\varepsilon_0,\qquad |\arg\varepsilon|\leqq\theta_0. \tag{1. 2}$$

We assume that the matrix $A(x,\varepsilon)$ is expressed by the asymptotic expansion such that

$$A(x,\varepsilon)\sim\sum_{r=0}^{\infty}A_r(x)\varepsilon^r,\qquad \varepsilon\to 0 \tag{1. 3}$$

is uniformly valid in $|x|\leqq x_0$ and $|\arg\varepsilon|\leqq\theta_0$. The coefficients $A_r(x)$ are then necessarily holomorphic in $|x|\leqq x_0$ (Wasow [12]).

Sibuya [8] has proved that the local asymptotic analysis, as $\varepsilon\to 0$, of such differential equations can be reduced to the study of the special case that $A(x,\varepsilon)$ satisfies the following hypothesis:

$A_0(0)$ is nilpotent and of a Jordan canonical form.

It will therefore be assumed, from now on, that $A_0(0)$ has this special property.

In this paper we investigate the case that $A_0(0)$ has n Jordan blocks. As $A_0(0)$ is nilpotent, which means that $A_0(0)=0$, the leading matrix $A_0(x)$ in (1. 3) must be of the form

$$A_0(x)=x^pG(x), \tag{1. 4}$$

with p a positive integer, $G(x)$ holomorphic at $x=0$, and $G(0)\neq 0$.

ASSUMPTION 1.

$$A_0(x)=x^pG(x),$$

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