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A SECOND THEOREM OF CONSISTENCY FOR ABSOLUTE SUMMABILITY BY DISCRETE RIESZ MEANS

BY T. PATI

1. 1. Definitions and notations. Let $\sum a_n$ be any given infinite series, and let $\{\lambda_n\}$ be a monotonic increasing sequence of positive numbers, tending to infinity with n. Let us write

$$A_\lambda(\omega)=A_\lambda^0(\omega)=\sum_{\lambda_n\leqq\omega} a_n,$$

$$A_\lambda^r(\omega)=\sum_{\lambda_n<\omega}(\omega-\lambda_n)^r a_n, \qquad r>0.$$

Let us write $R_\lambda^r(\omega)=A_\lambda^r(\omega)/\omega^r$, $r\geqq 0$. $\sum a_n$ is said to be absolutely summable by Riesz means of *type* λ_n and *order* r, or summable $|R, \lambda_n, r|$, $r\geqq 0$, if

$$R_\lambda^r(\omega)\in BV(k, \infty),\text{[1]}$$

where k is some finite positive number.[2] We say that $\sum a_n$ is absolutely summable by *discrete Riesz means of type* λ_n and order r, or summable $|R^*, \lambda_n, r|$, $r\geqq 0$, if

$$\{\Omega_n\}\equiv\{R_\lambda^r(\lambda_n)\}\in BV.\text{[3]}$$

By definition, summability $|R, \lambda_n, 0|$ and summability $|R^*, \lambda_n, 0|$ are the same as absolute convergence.

Let P and Q be any two methods of summability. Then, by '$P\subset Q$' we mean that any series which is summable P is also summable Q. By '$P\sim Q$' we mean that $P\subset Q$ as well as $Q\subset P$.

It is easily seen that

$$|R, \lambda_n, r|\subset|R^*, \lambda_n, r|, \ r\geqq 0.$$

Throughout, for any sequence $\{f_n\}$, we shall write $\Delta f_n=f_n-f_{n+1}$, and K will denote a positive constant, not necessarily the same at each occurrence.

1. 2. It is known that $|R, \lambda_n, 1|\sim|R^*, \lambda_n, 1|$.[4] For summability $|R, \lambda_n, r|$ the

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1) By '$f(x)\in BV(h, k)$' we mean that $f(x)$ is a function of bounded variation over (h, k).

2) Obrechkoff (4), (5).

3) By '$\{f_n\}\in BV$' we mean that $\sum_n|f_n-f_{n-1}|<\infty$.

4) A proof of this by the present author has been quoted in Iyer [2].