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# ON CERTAIN OPERATORS ASSOCIATED WITH TENSOR FIELDS

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*Dedicated to Professor Shisanji Hokari on his sixtieth birthday*

## Introduction.

The theory of differential concomitants has been developed by Frölicher, Nijenhuis [1], [2], [6], Schouten [9] and others.[1)]

Schouten [9] introduced a concomitant $[P, Q]$ formed with tensor fields $P$ and $Q$ of type $(p+1, 0)$ and $(q+1, 0)$ respectively, which is a tensor field of type $(p+q+1, 0)$.

Nijenhuis [6] introduced a concomitant $[S, T]$ formed with a vector $s$-form $S$ and a vector $t$-form $T$ which is a vector $(s+t)$-form. Frölicher and Nijenhuis [1], [2], also introduced a concomitant $[S, \omega]$ formed with a vector $s$-form $S$ and a scalar $t$-form $\omega$ which is a scalar $(s+t)$-form.

On the other hand it was found that, in the study of a differentiable manifold $M$ with an almost complex structure $F$, the tensor

$$\begin{aligned} N(X, Y) &= \frac{1}{2}[F, F](X, Y) \\ &= [FX, FY]-F[X, FY]-F[FX, Y]+F^2[X, Y] \end{aligned}$$

introduced by Nijenhuis [5] plays an important part. It is now well known [4] that it is necessary and sufficient for an almost complex manifold to be complex that the Nijenhuis tensor $N$ formed with $F$ vanishes identically.

Nijenhuis [6] proved also that

$$[F, N]=0, \qquad [N, N]=0$$

and it seemed that there is no concomitant formed only with $F$ and its partial derivatives and being essentially distinct from $N$.

However Walker [13] found a tensor field of type (1, 4) involving the second partial derivatives of an almost complex structure $F$.

Willmore [14] introduced a tensor field $W$ of type (1, 4)

$$W=[F \barwedge N, N]=[F, N \barwedge N]$$

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1) The numbers between brackets refer to the Bibliography at the end of the paper.