KŌDAI MATH. SEM. REP.
20 (1968), 374—384

AUTOMORPHISMS OF A FREE NILPOTENT ALGEBRA

BY MOTOICHI OKUZUMI

Let F be a finite dimensional nilpotent algebra over a field K with index of nilpotency ρ: $F\supset F^2\supset F^3\supset\cdots\supset F^{\rho-1}\supset F^\rho=0$. Let $u_1, u_2, \cdots, u_n$ be a system of generators of F such that u's are linearly independent over K modulo F^2. We shall call F a free nilpotent algebra if the generators $u_1, u_2, \cdots, u_n$ satisfy only relations $u_{i_1}u_{i_2}\cdots u_{i_\rho}=0$ $(1\leqq i_1, i_2, \cdots, i_\rho\leqq n)$; we shall denote it by $F=F(u_1, u_2, \cdots, u_n; \rho)$.

Let N be a nilpotent algebra over K with index of nilpotency ρ generated by n elements $a_1, a_2, \cdots, a_n$ and let F be as above. Then we can find a homomorphism φ of F onto N defined by $\varphi(u_i)=a_i$, so that N is isomorphic to the residue class ring $F/\mathfrak{p}$ where $\mathfrak{p}$ is the kernel of φ. Thus we may say that the study of nilpotent algebras can be reduced to that of free nilpotent algebras and their ideals.

In this note we shall consider a free nilpotent algebra and its automorphism groups. The first section is preliminary and we make some considerations about the relations between nilpotent algebras and free nilpotent algebras. In the second, we study automorphisms of a free nilpotent algebra. Throughout the note, we assume that the characteristic of the ground field K is 0, and algebras mean associative finite dimensional algebras over K.

1. Preliminaries. The following theorem is well known.

THEOREM 1. *Let N be a nilpotent algebra over a field K with index of nilpotency ρ, then N is generated by a system of elements $a_1, a_2, \cdots, a_n$ whicn form a basis of N modulo N^2. And any such system of elements generates N.*

We call such a system of elements a *minimal generating system of N*. From theorem 1, we get

COROLLARY. *Every nilpotent algebra N over K with index of nilpotency ρ is isomorphic to a residue class ring of a free nilpotent algebra F with index ρ by a two-sided ideal $\mathfrak{p}$ which is contained in F^2.*

If $a_1, a_2, \cdots, a_n$ generate N, the isomorphism mentioned in the above corollary is induced by the mapping of F onto N defined by

$$F\ni u_i\rightarrow a_i\in N, \qquad i=1, 2, \cdots, n.$$

If we take another minimal generating system $a_1', a_2', \cdots, a_n'$, we have the following

Received November 27, 1967; revised January 25, 1968.