

## DIFFERENTIAL GEOMETRY OF TANGENT BUNDLES OF ORDER 2

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*Dedicated to Professor Shisanji Hokari on his Sixtieth Birthday*

### § 0. Introduction.

The differential geometry of tangent bundles has been studied by Davies [13], Dombrowski [1], Kobayashi [15], Ledger [2], [3], [16], Morimoto [4], [5], Okumura [8], Sasaki [6], Tachibana [8], Tanno [9], Tondeur [10], the present authors [2], [3], [11], [13], [14], [15], [16], [17], [18] and others and that of cotangent bundles by Patterson [17], [18], Satô [7] and one of the present authors [12], [17], [18].<sup>1)</sup>

The purpose of the present paper is to study the differential geometry of tangent bundles of order 2, the tangent bundle of order 2  $T_2(M)$  of a differentiable manifold  $M$  being defined as the set of all 2-jets of  $M$  determined by mappings of the real line  $R$  into  $M$ .

In § 1, we define the tangent bundles of order 2 and induced coordinates in it and fix the notations used throughout the paper.

In § 2, we study the lifts of functions and two vector fields  $A$  and  $B$  existing a priori in  $T_2(M)$ .

§ 3 is devoted to the study of lifts of vector fields, 1-forms and derivations, and § 4 to the study of lifts of tensor fields and two linear mappings  $\alpha$  and  $\beta$ . In § 5, we give the local expressions of these lifts.

In § 6, we study in more detail the lifts of tensor fields of type (1, 1) and discuss lifts of torsion tensors and Nijenhuis tensors.

§ 7 is devoted to the study of lifts of affine connections and also of curvature tensor and torsion tensor of the connection.

We study lifts of infinitesimal transformations in § 8 and geodesics in  $T_2(M)$  in the last § 9.

### CONTENTS

- § 1. Tangent bundles of order 2.
- § 2. Lifts of functions.
- § 3. Lifts of vector fields, 1-forms and derivations.
- § 4. Lifts of tensor fields.
- § 5. Local expressions.

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1) The numbers in brackets [ ] refer to Bibliography at the end of the paper.