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EXISTENCE OF MAXIMAL ANALYTIC FUNCTIONS ON RIEMANN SURFACES

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1. Introduction. Given a Riemann surface W (open or closed), let $\mathcal{A}(W)$ and $\mathcal{M}(W)$ denote respectively the set of all single-valued analytic functions on W and the set of all single-valued meromorphic functions on W.

For any function $f\in\mathcal{A}(W)$ (or $g\in\mathcal{M}(W)$), take an arbitrary $\varphi\in\mathcal{A}(f(W))$ (or $\phi\in\mathcal{M}(f(W))$) and the composite function $\varphi\circ f$ (or $\phi\circ g$) still belongs to $\mathcal{A}(W)$ (or $\mathcal{M}(W)$). This fact suggests us to consider the indecomposable functions (i.e. impossible to be represented in the above-mentioned composite form) as, in a sense, fundamental for the surface W. In the present paper we shall be concerned with the existence theorem of such functions. Naturally, if φ is a one-to-one conformal map of $f(W)$ onto itself, then we have $f=\varphi\circ(\varphi^{-1}\circ f)$. So we have to speak of the indecomposability, always up to such trivial decompositions.

2. The reasoning being completely parallel for the case of meromorphic functions, in what follows we shall mostly confine ourselves to analytic functions on W.

DEFINITION. For two functions f, $g\in\mathcal{A}(W)$, if we have

$$g=\varphi\circ f \qquad \text{where} \qquad \varphi\in\mathcal{A}(f(W))$$

($f(W)$ being the image of W by f in the complex plane), we shall say: '*f majorizes g*' and we shall express this fact by $g\prec f$.

Obviously, '$f\prec g$ and $g\prec f$' is equivalent to '$f=\varphi\circ g$ and φ is a one-to-one conformal map of $g(W)$ onto itself'. So we may define an equivalence relation on $\mathcal{A}(W)$ by

$$f\sim g \Longleftrightarrow f\prec g \quad \text{and} \quad g\prec f \tag{2.1}$$

and the relation $\prec$ induces an order relation on the set of quivalence classes of $\mathcal{A}(W)$. Our problem is reduced to assure the existence of maximal equivalence classes with respect to this order relation.

DEFINITION. A function $f\in\mathcal{A}(W)$ is said to be *maximal* if f belongs to a maximal equivalence class.

Our final result is:

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