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INTEGRAL INEQUALITIES IN A COMPACT ORIENTABLE MANIFOLDS, RIEMANNIAN OR KÄHLERIAN

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Introduction. Obata [2] has recently obtained some integral inequalities satisfied by a function f in a compact orientable Riemannian manifold. In this paper, we study integral inequalities satisfied by a vector field in a compact orientable Riemannian manifold and in a compact Kählerian manifold.

§ 1. Integral inequalities in a compact orientable Riemannian manifold.

Let M be an n-dimensional compact orientable Riemannian manifold. We denote by d the operator which operates on a skew symmetric tensor of degree p, u: $u_{i_1\cdots i_p}$ and gives a skew symmetric tensor of degree $p+1$,

$$du:\quad \nabla_i u_{i_1\cdots i_p} - \nabla_{i_1} u_{i i_2\cdots i_p} \cdots - \nabla_{i_p} u_{i_1\cdots i_{p-1} i},$$

by δ the operator which operates on u and gives a skew symmetric tensor of degree $p-1$,

$$\delta u:\quad g^{ji}\nabla_j u_{i i_2\cdots i_p},$$

and by D the operator which operates on u and gives a skew symmetric tensor of degree $p+1$,

$$Du:\quad \nabla_i u_{i_1\cdots i_p} + \nabla_{i_1} u_{i i_2\cdots i_p} + \cdots + \nabla_{i_p} u_{i_1\cdots i_{p-1} i},$$

∇_j being the operator of covariant differentiation with respect to the Christoffel symbols $\{{}_j{}^h{}_i\}$ formed with the fundamental tensor g_{ji} of M. Furthermore we denote by Δ the operator $\delta d + d\delta$ and by $\square$ the operator $\delta D - D\delta$. For a vector u, we have

$$\Delta u:\quad g^{ij}\nabla_i\nabla_j u_h - K_h{}^i u_i,$$

and

$$\square u:\quad g^{ij}\nabla_i\nabla_j u_h + K_h{}^i u_i,$$

$K_h{}^i$ being the Ricci tensor. We define the global inner product of two tensors $a_{i_1\cdots i_p}$ and $b_{i_1\cdots i_p}$ of the same order p by

$$(a, b) = \frac{1}{p!}\int_M a_{i_1\cdots i_p} b^{i_1\cdots i_p} d\sigma,$$

$d\sigma$ being the volume element of the manifold M.

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