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ON THE GROUP OF (1, 1) CONFORMAL MAPPINGS OF AN OPEN RIEMANN SURFACE ONTO ITSELF

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1. The following theorem [4] is well known:

The number of (1, 1) *conformal mappings of a plane region bounded by* p ($\infty > p > 2$) *Jordan curves onto itself is finite.*

In the present paper we shall consider, instead of a plane region, an open Riemann surface W and shall give two sufficient conditions that W admits only a finite number of (1, 1) conformal mappings onto itself: namely,

THEOREM 1. *If* W *is an open Riemann surface which has* p ($\infty > p > 2$) *boundary elements in the sense of Kerékjártó-Stoilow, then the number of* (1, 1) *conformal mappings of* W *onto itself is finite.*

THEOREM 2. *If* W *is an open Riemann surface of genus* g ($\infty > g > 0$), *then the number of* (1, 1) *conformal mappings of* W *onto itself is finite.*

Theorem 1 may be regarded as an extension of the above theorem.

Further we shall consider an open Riemann surface which has precisely two boundary elements. In this case we shall exclude doubly connected planar surfaces from our investigation. There is a non-planar Riemann surface which has two boundary elements and which admits infinitely many (1, 1) conformal mappings onto itself. However we shall prove the following theorem.

THEOREM 3. *If* W *is an open Riemann surface which has two boundary elements and which is not planar, then the group of* (1, 1) *conformal mappings of* W *onto itself is finitely generated.*

More generally, let β_1 and β_2 be two boundary elements of an open Riemann surface W which has more than one boundary element and denote by $A(\beta_1, \beta_2)$ the group of (1, 1) conformal mappings φ of W onto itself which have the property that either

(1) $\varphi(p)$ tends to β_1, β_2 for p tends to β_1, β_2 respectively; or else

(2) $\varphi(p)$ tends to β_2, β_1 for p tends to β_1, β_2 respectively.

For such a group we have

THEOREM 4. *If* W *is an open Riemann surface which has more than one boundary element and which is not a doubly connected planar surface, than* $A(\beta_1, \beta_2)$

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