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CONJUGATE CLASSES OF ORISPHERICAL SUBALGEBRAS IN REAL SEMISIMPLE LIE ALGEBRAS

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Introduction.

Let $\mathfrak{g}$ be a real semisimple Lie algebra and let $\mathcal{G}$ be a Lie group whose Lie algebra is $\mathfrak{g}$. Let $g(t)=\exp(tX)$ be a one-parameter subgroup in $\mathcal{G}$ generated by X in $\mathfrak{g}$. We define an orispherical subgroup $\mathcal{Z}$ relative to X (or $g(t)$) as follows.

DEFINITION.

$$\mathcal{Z}=\left\{z \in \mathcal{G};\ \lim_{t\to\infty} g(t)zg(-t)=e\right\}.$$

$\mathcal{Z}$ is a connected closed subgroup of $\mathcal{G}$ [5], and its Lie algebras is equal to

$$\mathfrak{z}=\left\{Z\in\mathfrak{g};\ \lim_{t\to\infty} e^{t\,\mathrm{ad}\,X}Z=0\right\}.$$

This Lie algebra is called an orispherical subalgebra relative to X. I. M. Gel'fand and M. I. Graev [2] showed that these subgroups play an important role in the theory of group representations. Moreover, in the theory of unitary representations in homogeneous spaces with discrete stationary subgroups, Gel'fand and Pyatetskii-Shapiro [4] gave an effective process for isolating the discrete spectrum from the continuous spectrum. This process was the method of orispheres, the orbit of orispherical subgroups. In connection with this, the same authors [3] remarked and used the fact that, in the case of $\mathrm{SL}(n, R)$, there exist as many conjugate orispherical subgroups as there are representations of n in the form of a sum of positive summands $n=k_1+k_2+\cdots+k_s$.

In this note we wish to show that there is a one to one correspondence between conjugate classes of orispherical subalgebras and the set of faces of a Weyl chamber of $(\mathfrak{g}, \mathfrak{k})$, where $\mathfrak{k}$ is a maximal compactly imbedded subalgebra of $\mathfrak{g}$. (for the definitions, see below). Our method and results are somewhat similar to the case of Cartan subalgebras, but are more simple. We shall use frequently notations and results appeared in [6].

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