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ON $|C, 1|_k$ SUMMABILITY FACTORS OF FOURIER SERIES

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1. 1. Let $\sum a_n$ be a given infinite series with its n-th partial sum s_n, and let $t_n = t_n^0 = na_n$. By $\{\sigma_n^\alpha\}$ and $\{t_n^\alpha\}$ we denote the n-th Cesàro means of order α $(\alpha > -1)$ of the sequences $\{s_n\}$ and $\{t_n\}$ respectively. The series $\sum a_n$ is said to be absolutely summable (C, α) with index k, or simply summable $|C, \alpha|_k$ $(k \geqq 1)$, if

$$\sum n^{k-1} |\sigma_n^\alpha - \sigma_{n-1}^\alpha|^k < \infty. \tag{1.1.1}$$

Summability $|C, \alpha|_1$ is the same as summability $|C, \alpha|$. Since

$$t_n^\alpha = n(\sigma_n^\alpha - \sigma_{n-1}^\alpha),$$

condition (1. 1. 1) can also be written as

$$\sum \frac{|t_n^\alpha|^k}{n} < \infty. \tag{1.1.2}$$

A sequence $\{\lambda_n\}$ is said to be convex if $\Delta^2 \lambda_n \geqq 0$, $n = 1, 2, \cdots$, where $\Delta \lambda_n = \lambda_n - \lambda_{n+1}$ and $\Delta^2 \lambda_n = \Delta(\Delta \lambda_n)$.

1. 2. Let $f(t)$ be a periodic function with period 2π and integrable in the sense of Lebesgue over $(-\pi, \pi)$. Let the fourier series of $f(t)$ be given by

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{n=0}^{\infty} A_n(t),$$

where we can assume, without loss of generality, that $a_0 = 0$.

We shall use throughout this paper the following notations and identities:

$$\varphi(t) = \frac{1}{2} \{f(x+t) + f(x-t) - 2f(x)\},$$

$$D_n(t) = \frac{1}{2} + \cos t + \cos 2t + \cdots + \cos nt = \frac{\sin (n+1/2)t}{2 \sin (t/2)},$$

$$s_n(x) = \sum_{\nu=0}^{n} A_\nu(x) = \frac{1}{\pi} \int_0^\pi \{f(x+t) + f(x-t)\} D_n(t) dt,$$

$$s_n(x) - f(x) = \frac{1}{\pi} \int_0^\pi \{f(x+t) + f(x-t) - 2f(x)\} D_n(t) dt = \frac{2}{\pi} \int_0^\pi \varphi(t) D_n(t) dt,$$

and

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