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CONVERGENCE OF NORMAL OPERATORS

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The first purpose of the present paper is to show that elementary linear operator theory can be used to give an elegant proof of the fundamental existence theorem of principal functions corresponding to given normal operators L [17]. When L is defined by a limiting process a harmonic function may also be obtained by applying the main theorem to each approximating operator and forming a limit of resulting functions. It is important to know when these processes commute. We shall give a general criterion to this effect and show that it applies to operators L_0 and L_1. Earlier literature on normal operators and their applications is compiled in the Bibliography.

1. The q-lemma. We start with a slight sharpening of the q-lemma [17]:

LEMMA 1. *Let K be a compact subset of a Riemann surface W. There exists a positive constant $q<1$ such that all harmonic functions u on W satisfy the inequality*

$$q\inf_W u+(1-q)\max_K u\leqq u|K\leqq(1-q)\min_K u+q\sup_W u. \tag{1}$$

Proof. Harnack's inequality for positive harmonic functions v in the unit disk reads

$$\frac{1-|z|}{1+|z|}v(0)\leqq v(z)\leqq\frac{1+|z|}{1-|z|}v(0).$$

An easy consequence is that to any compact set K in a Riemann surface W there corresponds a constant $c>0$ such that

$$c^{-1}\leqq\frac{v(\mathrm{P})}{v(\mathrm{Q})}\leqq c \tag{2}$$

for all points P and Q in K and all positive harmonic functions v. To see this note first that K may be assumed to be connected, thanks to the existence of an exhaustion for W. K can be covered by a finite number n of parametric disks $\mathcal{V}_i$ with centers V_i such that the subdisks $\mathcal{V}_i'$ corresponding to $\{z: |z|<1/2\}$ also form an open cover of K. By Harnack's inequality $1/3<v(\mathrm{P}_i)/v(\mathrm{Q}_i)<3$ for $\mathrm{P}_i\in\mathcal{V}_i'$, and conse-

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