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ON A CONTINUITY LEMMA OF EXTREMAL LENGTH AND ITS APPLICATIONS TO CONFORMAL MAPPING

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§ 1. Introduction.

1. The continuity of the extremal length of a curve family joining two disjoint compact sets in the plane with respect to their exhaustion was first discussed by Wolontis [10]. Later Strebel [7] showed the continuity for the Riemann surface and its two sets of compact boundary components (in the Stoilow compactification), and recently Marden and Rodin [4] generalized it for a wider class of curves. In the present paper we shall show the continuity of the extremal length with respect to increasing curve families. Such a property was already discussed for particular curve families in a problem of conformal mappings by Marden and Rodin [4], but we state the continuity in a general form.

As an application of the continuity lemma, we shall discuss a problem of conformal mapping from a domain onto a slit rectangle. The problem was first treated by Grötzsch [2] in the case of finite connectivity. In our former paper [8] we constructed a slit rectangle mapping function for a domain whose outer boundary was isolated. In the present paper we shall show that a plane domain with a preassigned boundary component given four distinct curves (vertices) can be mapped onto a horizontally slit rectangle with possible horizontal incisions, if the extremal length of the family of curves joining one pair of edges corresponding to vertical sides is finite.

§ 2. Preliminary.

2. We sum up some known results for extremal lengths. Let R be a Riemann surface and let Γ be a family of curves on R. We mean by a curve a collection of at most countable open connected arcs whose member is locally rectifiable. Let $\rho(z)|dz|$ be a nonnegative measurable metric. We call ρ *measurable* on Γ, if the integral of ρ along each $\gamma\in\Gamma$ exists. A metric ρ is called *admissible*, if it is measurable and its integral along each $\gamma\in\Gamma$ is not less than one. An admissible class, denoted by $P(\Gamma)$, is the collection of all admissible metrics. The closure of the intersection of $P(\Gamma)$ with the l_2-space of the metrics with finite norm is called a *generalized* admissible class and written by $P^*(\Gamma)$. The module of Γ is defined by

$$\inf_{\rho\in P(\Gamma)}\iint\rho^2 dxdy=\inf_{\rho\in P(\Gamma)}||\rho||^2$$

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