ON A CONTINUITY LEMMA OF EXTREMAL LENGTH AND ITS APPLICATIONS TO CONFORMAL MAPPING

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§ 1. Introduction.

1. The continuity of the extremal length of a curve family joining two disjoint compact sets in the plane with respect to their exhaustion was first discussed by Wolontis [10]. Later Strebel [7] showed the continuity for the Riemann surface and its two sets of compact boundary components (in the Stoliow compactification), and recently Marden and Rodin [4] generalized it for a wider class of curves. In the present paper we shall show the continuity of the extremal length with respect to increasing curve families. Such a property was already discussed for particular curve families in a problem of conformal mappings by Marden and Rodin [4], but we state the continuity in a general form.

As an application of the continuity lemma, we shall discuss a problem of conformal mapping from a domain onto a slit rectangle. The problem was first treated by Grötzsch [2] in the case of finite connectivity. In our former paper [8] we constructed a slit rectangle mapping function for a domain whose outer boundary was isolated. In the present paper we shall show that a plane domain with a preassigned boundary component given four distinct curves (vertices) can be mapped onto a horizontally slit rectangle with possible horizontal incisions, if the extremal length of the family of curves joining one pair of edges corresponding to vertical sides is finite.

§ 2. Preliminary.

2. We sum up some known results for extremal lengths. Let \( R \) be a Riemann surface and let \( \Gamma \) be a family of curves on \( R \). We mean by a curve a collection of at most countable open connected arcs whose member is locally rectifiable. Let \( \rho(z)|dz| \) be a nonnegative measurable metric. We call \( \rho \) measurable on \( \Gamma \), if the integral of \( \rho \) along each \( \gamma \in \Gamma \) exists. A metric \( \rho \) is called admissible, if it is measurable and its integral along each \( \gamma \in \Gamma \) is not less than one. An admissible class, denoted by \( P(\Gamma) \), is the collection of all admissible metrics. The closure of the intersection of \( P(\Gamma) \) with the \( l_\infty \)-space of the metrics with finite norm is called a generalized admissible class and written by \( P^*(\Gamma) \). The module of \( \Gamma \) is defined by

\[
\inf_{\rho \in P(\Gamma)} \int \rho^2 \, dx dy = \inf_{\rho \in P(\Gamma)} \|\rho\|^2
\]

Received July 6, 1966.