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SURFACES OF CURVATURES $\lambda=\mu=0$ IN E^4

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Introduction. A complete surface of Gaussian curvature $G=0$ in Euclidean space E^3 of dimension 3 is a cylinder. This fact was proved by W. S. Massey. The purpose of this paper is to prove the following theorem:

THEOREM. *A complete surface M^2 in Euclidean space E^4 of dimension 4 with the curvatures $\lambda=\mu=0$ is a cylinder.*

The definition of a cylinder in an Euclidean space is given by the following: Through each point of a curve on it, there passes a straight line which has the constant direction and the curve is not equal to one of these straight lines. The method of the proof is due to the idea of the Frenet-frames given by Professor Ōtsuki. In §1, we study the local properties of M^2 in E^4 without completeness. A global study of complete surfaces of the curvatures $\lambda=\mu=0$ is given in §2 with the aid of the universal covering space. The above theorem will be proved in this section. The author expresses his deep gratitude to Professor Ōtsuki who encouraged him and gave him a lot of useful suggestions.

§1. In the following we consider 2 dimensional, connected, oriented and class C^4 Riemannian manifold M^2 immersed in E^4 with the principal and secondary curvatures $\lambda=\mu=0$. By the definition of the Frenet-frame (p, e_1, e_2, e_3, e_4) for any surface M^2 in E^4, we have the following:

$$dp=e_1\omega_1+e_2\omega_2, \qquad de_A=\Sigma\omega_{Aj}e_j+\omega_{A3}e_3+\omega_{A4}e_4, \qquad A=1,2,3,4, \qquad j=1,2, \tag{1.1}$$

$$\omega_{13}\wedge\omega_{24}+\omega_{14}\wedge\omega_{23}=0, \tag{1.2}$$

$$\omega_{13}\wedge\omega_{23}=\lambda\omega_1\wedge\omega_2, \qquad \omega_{14}\wedge\omega_{24}=\mu\omega_1\wedge\omega_2, \tag{1.3}$$

$$\lambda+\mu=G, \qquad \lambda\geqq\mu, \tag{1.4}$$

where ω_1, ω_2 and $\omega_{12}=-\omega_{21}$ are the basic forms and the connection form of M^2 respectively, λ and μ are the principal and secondary curvatures of the surface respectively, and G is the Gaussian curvature of M^2. In our case we cannot define the uniquely determined Frenet-frame, but we suitably take such a frame (p, e_1, e_2, e_3, e_4) from the first. Putting $\omega_{ir}=\Sigma A_{rij}\omega_j$ where $i, j=1, 2$, $r=3, 4$ and $A_{rij}=A_{rji}$ we get by the hypothesis

$$\operatorname{rank}(A_{3ij})\leqq 1, \qquad \operatorname{rank}(A_{4ij})\leqq 1. \tag{1.5}$$

Then we could define the two sets:

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