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ON WIENER'S FORMULA FOR STOCHASTIC PROCESSES

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1. Let $\mathcal{E}(t)$ $(-\infty<t<\infty)$ be a weakly stationary stochastic process with the spectral representation:

$$\mathcal{E}(t)=\int_{-\infty}^{\infty} e^{it\lambda}dZ(\lambda), \tag{1}$$

and let

$$X(t)=f(t)+\mathcal{E}(t), \tag{2}$$

where $f(t)$ is a numerical valued function. Consider the stochastic integral

$$\int_{-\infty}^{\infty} X(t)aK(at)dt$$

with $K(t)\in L_1(-\infty,\infty)$. Kawata [1] has shown that under some conditions on $K(t)$ and $f(t)$ we have the following Wiener type formula:

$$\underset{a\to 0}{\text{l.i.m.}}\int_{-\infty}^{\infty} X(t)e^{-i\xi t}aK(at)dt=[M_\xi+Z(\xi+0)-Z(\xi-0)]\int_{-\infty}^{\infty} K(t)dt, \tag{3}$$

where ξ is a real constant and

$$M_\xi=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} f(t)e^{-i\xi t}dt.$$

The purpose of this paper is to prove the similar formula for the more general class of stochastic processes.

2. We state first the following

LEMMA. *Let $\{f_\lambda(\cdot)\}_{\lambda\in\Lambda}$ be a class of functions defined on* $(0,\infty)$. *If*
(i) *$K(x)$ is absolutely continuous in every finite interval,*
(ii) *$|x^2K(x)|<H$, $K(x)\in L_1(0,\infty)$, H being a constant,*
(iii) *$\frac{1}{T}\int_0^T |f_\lambda(t)|dt\leqq G$, G being a constant independent of λ and T, and*
(iv) *$\lim_{T\to\infty}\frac{1}{T}\int_0^T f_\lambda(t)dt=M_\lambda$, uniformly in $\lambda\in\Lambda$,*

then

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