DIFFERENTIAL GEOMETRY IN TANGENT BUNDLE

By KENTARO YANO AND SHIGERU ISHIHARA

The differential geometry of tangent bundle of a Riemannian manifold has been studied by Sasaki [4] and that of a Finslerian manifold by Yano and Davies [8].

It is now well known [1], [3], [5], [8] that the tangent bundle of a differentiable manifold with a linear connection admits an almost complex structure and that the integrability condition of the almost complex structure is the vanishing of torsion and curvature tensors of the linear connection. Yano and Davies [8] used this fact in their study of the tangent bundle of a Finslerian manifold.

A linear connection in an n-dimensional differentiable manifold may be defined as an n-dimensional distribution, transversal to the fibre and invariant by all right translations in the principal fibre bundle associated with the tangent bundle.

On the other hand, what we call a non-linear connection is defined as an n-dimensional distribution, transversal to the fibre and invariant by all dilatations in the tangent bundle, and consequently a linear connection is of course a non-linear connection.

The main purpose of the present paper is to study the differential geometry of tangent bundle of a differentiable manifold with a non-linear connection.

In § 1, we define the non-linear connection as a distribution in the tangent bundle and in § 2 we introduce what we call adapted frame which is very suitable for the study of differential geometry of tangent bundle of a manifold with a non-linear connection.

§ 3 is devoted to the study of the three kinds of lifts, horizontal, vertical and complete.

In § 4, we show that the tangent bundle of a manifold with a non-linear connection admits an almost complex structure and study the integrability condition of the almost complex structure.

In § 5, we study what we call restricted tensor fields which played an important rôle in the classical theory of manifolds with a non-linear connection and in the theory of Finslerian manifolds.

Since the tangent bundle of a manifold with non-linear connection admits an almost complex structure, we can talk about almost analytic vector fields in the tangent bundle. We study in § 6 these vector fields which could be obtained as lifts of vector fields in the underlying manifold.

We then in § 7 introduce a linear connection in tangent bundle of the tangent bundle of the manifold which has special importance.

In the last § 8, we shall discuss properties of curves which are obtained as lifts

Received January 24, 1966.