ON THE AUTOMORPHISM GROUPS OF f-MANIFOLDS

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Introduction.

In 1933, H. Cartan [4][1] proved that the group of all complex analytic transformations of a bounded domain in C^n is a Lie transformation group. As a matter of fact, the group of differentiable transformations on a differentiable manifold leaving a certain geometric structure invariant is often a Lie transformation group. The problem has been studied by many authors. Recently, Chu and Kobayashi [5] have summarized these known results in the chronological order and given systematic proofs. On the other hand, Ruh [11] has obtained a condition under which the group of differentiable transformations leaving a G-structure invariant on a compact differentiable manifold is a Lie transformation group.

The purpose of the present paper is to prove that the automorphism group of a compact f-manifold of some kind is a Lie transformation group (Theorem in § 2). We shall give the proof in § 4.

§ 1. (f, g)-manifolds.

Let V be an n-dimensional connected differentiable manifold of class C^∞. If there exists a non-null tensor field f of type (1, 1) and of class C^∞ satisfying

$$f^3+f=0, \tag{1.1}$$

and if the rank of f is constant everywhere and is equal to r, then we call such a structure an *f-structure of rank r* (Yano [14]). We call a differentiable manifold admitting an f-structure an *f-manifold*. We put

$$l=-f^2, \qquad m=f^2+1, \tag{1.2}$$

where 1 denotes the unit tensor, then we have

$$l+m=1, \qquad l^2=l, \qquad m^2=m, \qquad lm=ml=0. \tag{1.3}$$

These equations mean that the operators l and m applied to the tangent space at each point of the manifold are complementary projection operators. Thus, there exist in the manifold complementary distributions L and M corresponding to the projection operators l and m respectively. When the rank of f is equal to r, L is r-dimensional and M is $(n-r)$-dimensional.

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1) Numbers in brackets refer to the bibliography at the end of the paper.