ON A SIMPLIFIED METHOD OF THE ESTIMATION OF THE CORRELOGRAM FOR A STATIONARY GAUSSIAN PROCESS, III

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§ 1. Introduction.

In this paper we shall deal with a simplified method for the estimation of the correlogram for a stationary process.

Let $X(n)$ be a real-valued stationary process with discrete time parameter n. We assume $EX(n)=0$. We put

$$EX(n)^2=\sigma^2, \qquad EX(n)X(n+h)=\sigma^2\rho_h,$$

and we consider to estimate the correlogram ρ_h.

In the previous papers [4], [5], we discussed a simplified method for the estimation of the correlogram when σ^2 is known. But in the present paper, we discuss the case when σ^2 is unknown. For simplicity, let us assume the process $X(n)$ to be observed at $n=1, 2, \cdots, N, \cdots, N+h$.

Usually, in order to estimate the correlogram ρ_h, we use the estimate

$$\tilde{\Gamma}_h=\frac{\sum_{n=1}^{N} X(n)X(n+h)}{\sum_{n=1}^{N} X(n)^2}.$$

Now we shall modify the estimate $\tilde{\Gamma}_h$. The essential part of our modification is to replace $X(n)X(n+h)$ by $X(n)\,\mathrm{sgn}\,(X(n+h))$, where $\mathrm{sgn}\,(y)$ means $1, 0, -1$ correspondingly as $y>0$, $y=0$, $y<0$. The new estimate is

$$\Gamma_h=\frac{\sum_{n=1}^{N} X(n)\,\mathrm{sgn}\,(X(n+h))}{\sum_{n=1}^{N} |X(n)|}.$$

This new estimate Γ_h may be considered as follows. We make a nonlinear operation on the input $X(n)$ and assume that the output is $Y(n)=\mathrm{sgn}\,(X(n))$. Then, the estimate Γ_h consists of the cross-correlation of the input $X(n)$ and the output $Y(n)$.

We shall show below that when $X(n)$ is a Gaussian process satisfying some conditions, the estimate Γ_h is an asymptotically unbiased estimate of the correlogram

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