SURFACES IN THE 4-DIMENSIONAL EUCLIDEAN SPACE ISOMETRIC TO A SPHERE

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In [3], the author introduced some kinds of curvatures and torsion form for surfaces in a higher dimensional Euclidean space. These curvatures are linearly dependent on the Gaussian curvature and carry out the same rôles of the curvature and the torsion of a curve in the 3-dimensional Euclidean space with the torsion form. In the present paper, the author will investigate the isometric immersions of the two dimensional sphere into the 4-dimensional Euclidean space with constant curvatures.

§ 1. Preliminaries.

Let M^2 be a 2-dimensional oriented Riemannian C^∞-manifold with an isometric immersion

$$x: \ M^2 \to E^4$$

of M^2 into a 4-dimensional Euclidean space E^4. Let $F(M^2)$ and $F(E^4)$ be the bundles of orthonormal frames of M^2 and E^4 respectively. Let B be the set of elements $b=(p, e_1, e_2, e_3, e_4)$ such that $(p, e_1, e_2) \in F(M^2)$ and $(x(p), e_1, e_2, e_3, e_4) \in F(E^4)$ whose orientations is coherent with the one of E^4, identifying e_i with $dx(e_i)$, $i=1, 2$. $B \to M^2$ may be considered as a principal bundle with the fibre $O(2) \times SO(2)$. Let

$$\tilde{x}: \ B \to F(E^4)$$

be the mapping naturally defined by $\tilde{x}(b)=(x(p), e_1, e_2, e_3, e_4)$. Let B_ν be the set of elements (p, e) such that $p \in M^2$ and e is a unit normal vector to the tangent plane $dx(T_p(M^2))$ at $x(p)$. $B_\nu \to M^2$ is the so-called normal circle bundle of M^2 in E^4 whose fibre at p is denoted by S^1_p. Let S^3_0 be the unit 3-sphere in E^4 with the origin as its center. Let

$$\tilde{\nu}: \ B \to S^3_0$$

be the mapping defined by $\tilde{\nu}(p, e)=e$.

We have the differential forms ω_1, ω_2, ω_{12}, ω_{13}, ω_{14}, ω_{23}, ω_{24}, ω_{34} on B derived from the basic forms and the connection forms on $F(E^4)$ of the Euclidean space E^4 through $\tilde{x}$ as follows:

Received September 27, 1965.