$|\bar{N}, p_n|$ SUMMABILITY FACTORS OF INFINITE SERIES

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1. 1. Let $\sum a_n$ be a given infinite series with s_n as its n-th partial sum. Also let $\{p_n\}$ be a sequence of positive real constants such that P_n tends to infinity with n, where $P_n=\sum_{\nu=0}^{n} p_\nu$. We write

$$t_n=\frac{1}{P_n}\sum_{\nu=0}^{n} p_\nu s_\nu. \tag{1.1.1}$$

The series $\sum a_n$ is said to be absolutely summable $(\bar{N}, p_n)$ or, simply summable $|\bar{N}, p_n|$, if the sequence $\{t_n\}$ is of bounded variation.

If for some finite s

$$\sum_{\nu=1}^{n} |s_\nu - s| p_\nu = o(P_n),$$

as $n\to\infty$, then $\sum a_n$ is said to be strongly summable $(\bar{N}, p_n)$ or, simply summable $[\bar{N}, p_n]$. If

$$\sum_{\nu=1}^{n} |s_\nu| p_\nu = O(P_n),$$

as $n\to\infty$, then $\sum a_n$ is said to be bounded $[\bar{N}, p_n]$.

Writing $p_n=1/n$ in the above definitions we get summability $|R, \log n, 1|$,[1] summability $[R, \log n, 1]$ and bounded $[R, \log n, 1]$ respectively.

1. 2. Suppose $\sum a_n$ is summable $|\bar{N}, p_n|$. Then, since

$$s_{n+1}p_{n+1}=t_{n+1}P_{n+1}-t_nP_n,$$

we have

$$\begin{aligned}\sum_1^m |s_{n+1}| p_{n+1} &= \sum_1^m |\Delta t_n P_n| \\ &\leqq \sum_1^m P_n |\Delta t_n| + \sum_1^m |t_{n+1}| |\Delta P_n| \\ &= O(P_m) + O\left(\sum_1^m |\Delta P_n|\right)\end{aligned}$$

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1) Summability $|R, \log n, 1|$ is equivalent to the summability $|\bar{N}, 1/n|$.