ON A MATCHING METHOD FOR A LINEAR ORDINARY DIFFERENTIAL EQUATION CONTAINING A PARAMETER, II

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§ 1. Introduction.

In this paper, we consider the asymptotic behavior of the solution of a linear ordinary differential equation of the form

$$\varepsilon^h \frac{dy}{dx} = A(x, \varepsilon) y \tag{1.1}$$

as the parameter ε tends to zero. Here we suppose that

1) h is a positive integer;

2) x and ε are a complex variable and a complex parameter respectively;

3) y is an n-dimensional column vector;

4) $A(x, \varepsilon)$ is an n-by-n matrix function holomorphic and bounded in the domain of the x, ε space defined by the inequalities,

$$|x| \leqq x_0 < 1, \qquad 0 < |\varepsilon| \leqq \varepsilon_0, \qquad |\arg \varepsilon| \leqq \delta_0; \tag{1.2}$$

5) when ε tends to zero in the domain

$$0 < |\varepsilon| \leqq \varepsilon_0, \qquad |\arg \varepsilon| \leqq \delta_0, \tag{1.3}$$

$A(x, \varepsilon)$ admits for $|x| \leqq x_0$ a uniform asymptotic expansion in powers of ε:

$$A(x, \varepsilon) \simeq \sum_{\nu=0}^{\infty} A^{(\nu)}(x) \varepsilon^\nu, \tag{1.4}$$

where the coefficients $A^{(\nu)}(x)$ are n-by-n matrices whose components are functions holomorphic and bounded for $|x| \leqq x_0$;

6) the matrix $A(x, \varepsilon)$ has the form

$$A(x, \varepsilon) = \begin{bmatrix} A_{11}(x, \varepsilon) & & & 0 \\ A_{21}(x, \varepsilon) & A_{22}(x, \varepsilon) & & \\ \cdots & & \ddots & \\ A_{p1}(x, \varepsilon) & \cdots & & A_{pp}(x, \varepsilon) \end{bmatrix}, \tag{1.5}$$

where $A_{jk}(x, \varepsilon)$ are n_j-by-n_k matrices $(j, k = 1, \cdots, p)$;

7) in particular, each of the matrices $A_{ii}(x, \varepsilon)$ has the form

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