NORMAL STRUCTURE f SATISFYING $f^3+f=0$

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A structure on an n-dimensional differentiable manifold given by a non-null tensor field f of constant rank r satisfying $f^3+f=0$ is called an *f-structure* [2, 6, 7].[1] If $n=r$, then an f-structure gives an almost complex structure of the manifold and $n=r$ is necessarily even. If the manifold is orientable and $n-1=r$, then an f-structure gives an almost contact structure of the manifold and n is necessarily odd.

Sasaki and Hatakeyama [4] have introduced the notion of normality in the study of almost contact structure and characterized the normal almost contact structure by the vanishing of a tensor field constructed from the structure. On the other hand, it is well known [1, 5, 9] that an almost complex structure in the tangent bundle is determined by giving a linear connection in the tangent bundle. The almost complex structure in the tangent bundle is complex if and only if the linear connection determining the almost complex structure is locally flat [5].

When an n-dimensional manifold V admits a non-null f-structure f of rank r such that $n-r\geqq 1$, there exist two distributions L and M corresponding to the projection operators $l=-f^2$ and $m=f^2+1$ respectively. The operator f operating on the tangent bundle $T(V)$ of the manifold V acts as an almost complex structure on the distribution L and as a null-operator on the distribution M. It is now well known [1, 5, 9] that an almost complex structure is determined in the tangent bundle when a linear connection is given in the tangent bundle. By a similar device as that used in the study of almost complex structure in the tangent bundle, we shall show in § 3 of the present paper that an almost complex structure is determined in the vector bundle $M(V)$ by giving a connection ω in the vector bundle $M(V)$, $M(V)$ being the vector bundle consisting of all tangent vectors belonging to the distribution M.

When the almost complex structure in the vector bundle $M(V)$ is complex analytic, we say that the f-structure f is *normal* with respect to the given connection ω. We shall prove in § 5 that the f-structure f is normal with respect to a connection ω given in the vector bundle $M(V)$ if and only if the connection ω is of zero curvature and a tensor field constructed from f and ω vanishes identically (Theorem 2). The notion of normal f-structure seems to be very useful in study of certain submanifolds immersed in an almost complex space (cf. [8]).

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1) The numbers between brackets refer to the Bibliography at the end of the paper.