ON GALOIS CONDITIONS IN DIVISION ALGEBRAS

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1. Introduction.

A division subring A of a division ring D is said to be Galois in D (and D is Galois over A), if A is the set of fixed elements of a group of automorphisms acting in D. When that is so, as commutative case, there is one to one correspondence between a division subring B of D over A and a closed group H of automorphisms of D with finite reduced order. And, in commutative case, we know that the necessary and sufficient conditions for D to be Galois over A are: D is finite, separable and normal over A. Jacobson, developing Galois theory in division rings, had shown that it is an unsolved problem to determine conditions on a division subring A of D in order that there exists a closed group G of finite reduced order whose ring of the fixed elements is A. And he had proved the following result which is in essence due to Teichmüller:

If Z_0 is a subfield of the center Z of a division ring D and $[D:Z]$ is finite, then necessary and sufficient conditions that there exists a closed group G of automorphisms whose set of fixed elements is Z_0 are

1) Z is separable and normal over Z_0 and
2) every automorphism of the Galois group of Z over Z_0 can be extended to an automorphism of D.

In the present paper we shall derive conditions for D to be Galois over its division subring A in the case of finite dimension over the center. In the followings we assume that the center of D has an infinite number of elements.

2. Central elements in a division ring.

In the followings we denote by $V_S(A)$ the set of all elements of S which are commutative with every element of A. Then,

Lemma 1. *If R is a ring with unit element e and center Z, and A is a subring of R containing e, then*

$$V_R(A)=V_R(A,Z),$$

where (A,Z) is the ring generated by A and Z.

Proof. It is evident that $V_R(A,Z)$ is contained in $V_R(A)$. If c is any element of $V_R(A)$ and a is any element of (A,Z), then

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