ON THE EXISTENCE OF ANALYTIC MAPPINGS, II

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1. Let $G(z)$ and $g(z)$ be two entire functions having no zero other than an infinite number of simple zeros, respectively. Let R and S be two ultrahyperelliptic surfaces defined by two equations $y^2=G(x)$ and $y^2=g(x)$, respectively. In our previous paper [3] we offered a conjectural problem: *Is the order ρ_G of G an integral multiple of the order ρ_g of g, when there is an analytic mapping φ from R into S?* As we remarked there, in this problem we should assume that $\rho_G<\infty$ and $0<\rho_g<\infty$ and further suitable normalizations on G and g are done. Let G_c and g_c be two canonical products having the same zeros with the same multiplicities as those of G and g, respectively. In this paper an analytic mapping means a non-trivial one.

THEOREM 1. *Assume that $\rho_{G_c}<\infty$ and $0<\rho_{g_c}<\infty$ and that there exists an analytic mapping φ from R into S. Then ρ_{G_c} is an integral multiple of ρ_{g_c}.*

This is somewhat effective criterion for the non-existence of an analytic mapping from R into S. Theorem 1 can be stated in the following form:

Assume that $\rho_{N(r;0,G)}<\infty$ and $0<\rho_{N(r;0,g)}<\infty$ and that there exists an analytic mapping φ from R into S. Then the former one is an integral multiple of the latter one.

2. To prove theorem 1 we need an elegant theorem due to Valiron [7]. We can state his result in the following manner.

Let $h(z)$ be an entire function satisfying one of the following conditions:
(a) *$h(z)$ has a finite order;*
(b) *There is a suitable number $\lambda>1$ satisfying*

$$\lim_{r\to\infty}\frac{\log V(r^\lambda)}{V(r)}=0,\quad V(r)=\log M(r),\quad M(r)=\max_{|z|\leqq r}|h(z)|.$$

Then the equation $h(z)=w$ has at least one solution z in the annulus

$$M^{-1}(|w|)\leqq|z|\leqq M^{-1}(|w|)^{1+\alpha}$$

for an arbitrary small positive number α, if $|w|$ is sufficiently large, $|w|>A(\alpha)$.

As Valiron remarked, (b) implies (a) and (b) is satisfied by a quite wide class of entire functions, which contains some entire functions of infinite order. He also gave another theorem which is more precise and applicable than the above.

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