ON A MATCHING METHOD FOR A LINEAR ORDINARY DIFFERENTIAL EQUATION CONTAINING A PARAMETER, I

By Toshihiko Nishimoto

§1. Introduction.

Let a linear ordinary differential equation of the form

$$\varepsilon^{h}\frac{dy}{dx}=A(x,\varepsilon)y, \tag{1.1}$$

be given, where h is a positive integer, x is a complex independent variable, ε is a complex parameter, and $A(x, \varepsilon)$ is an n-by-n matrix of the form

$$A(x,\varepsilon)=\begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ 0 & & & 1 \\ P_n(x,\varepsilon), & P_{n-1}(x,\varepsilon), & \cdots, P_2(x,\varepsilon), & 0 \end{bmatrix} \tag{1.2}$$

The functions $P_k(x, \varepsilon)$ $(k=2, \cdots, n)$ are holomorphic in the domain of the x, ε-space defined by the inequalities,

$$|x|\leqq x_0<1, \qquad 0<|\varepsilon|\leqq\varepsilon_0, \qquad |\arg \varepsilon|\leqq\delta_0 \tag{1.3}$$

and each of the functions $P_k(x, \varepsilon)$ has a uniformly asymptotic expansion in powers of ε such that

$$P_k(x,\varepsilon)\simeq\sum_{\nu=0}^{\infty}P_{k\nu}(x)\varepsilon^{\nu} \tag{1.4}$$

as ε tends to zero in the domain (1. 3) with holomorphic coefficients $P_{k\nu}(x)$:

$$P_{k\nu}(x)=\sum_{\mu=m_{k\nu}}^{\infty}x^{\mu}P_{k\nu\mu}, \quad P_{k\nu m_{k\nu}}\neq 0, \tag{1.5}$$

where $P_{k\nu\mu}$ are constant and $m_{k\nu}$ are non negative integers. Suppose that $m_{k0}\geqq 1$ for $k=2, \cdots, n$, then the equation (1. 1) has a turning point at the origin. We consider here the asymptotic behavior of solutions of equation (1. 1) in the neighborhood of the turning point as ε tends to zero. When $A(x, \varepsilon)$ is a 2-by-2 matrix, it was treated in this author's previous paper [2], and in this paper we shall generalize that results to the equations of higher order.

Assume that

Received June 18, 1965.