

# ON THE EXISTENCE OF ANALYTIC MAPPINGS BETWEEN TWO ULTRAHYPERELLIPTIC SURFACES

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**§ 1. Introduction.** Let  $R$  and  $S$  be two ultrahyperelliptic surfaces defined by two equations  $y^2=G(z)$  and  $w^2=g(w)$ , respectively, where  $G$  and  $g$  are two entire functions having no zero other than an infinite number of simple zeros. Then one of the authors [6], [7] established the following perfect condition for the existence of analytic mappings from  $R$  into  $S$ .

**THEOREM A.** *If there exists an analytic mapping  $\varphi$  from  $R$  into  $S$ , then there exists a pair of two entire functions  $h(z)$  and  $f(z)$  satisfying an equation*

$$f(z)^2G(z)=g\circ h(z)$$

*and vice versa.*

Let  $\mathfrak{M}(R)$  be a family of non-constant meromorphic functions on  $R$ . Let  $f$  be a member of  $\mathfrak{M}(R)$ . Let  $P(f)$  be the number of Picard's exceptional values of  $f$ , which we say  $\alpha$  a Picard's value of  $f$  when  $\alpha$  is not taken by  $f$  on  $R$ . Let  $P(R)$  be a quantity defined by

$$\sup_{f \in \mathfrak{M}(R)} P(f)$$

(cf. [4]). Let  $P(S)$  be the corresponding quantity attached to  $S$ .

In the present paper we shall give a perfect condition for the existence of analytic mappings from  $R$  into  $S$  in a case of  $P(R)=P(S)=4$ , which is more direct than theorem A, and shall give some characterizations of the surfaces  $R$  with  $P(R)=3$  by the forms of defining functions  $G$ .

By a characterization, which was given in [5], of  $R$  with  $P(R)=4$  by  $G(z)$  we can put

$$(1.1) \quad \begin{aligned} F(z)^2G(z) &= (e^{H(z)} - \gamma)(e^{H(z)} - \delta), & H(z) &\equiv \text{const.}, \\ H(0) &= 0, & \gamma\delta(\gamma - \delta) &\neq 0 \end{aligned}$$

with two suitable entire functions  $F$  and  $H$  and two constants  $\gamma$  and  $\delta$ . Similarly if  $P(S)=4$ , we can put

$$(1.2) \quad \begin{aligned} f(w)^2g(w) &= (e^{L(w)} - \gamma')(e^{L(w)} - \delta'), & L(w) &\equiv \text{const.}, \\ L(0) &= 0, & \gamma'\delta'(\gamma' - \delta') &\neq 0 \end{aligned}$$

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