ON A CERTAIN FUNCTIONAL-DIFFERENTIAL INEQUALITY

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Introduction.

Recently, the method using the relations described by some inequalities has been applied to the uniquenes problem for certain functional equations. For example, Nickel [5] has considered a functional equation including an operator T such that

$$(1) \qquad F(t, x', x, Tx)=0,$$

and obtained various criteria for the uniqueness of solutions of (1). If the operator T will be defined suitably, (1) will yield various types of equations. For example, if $F(t, x, y, z)$ is of the form such that

$$F(t, x, y, z)=y-g(t, x)-z,$$

and if T is defined by

$$Tx=\int_0^t K(t, s, x(s))ds,$$

(1) is reduced to an integro-differential equation

$$x'=g(t, x)+\int_0^t K(t, s, x(s))ds.$$

Hence, the results in [5] will be applicable to the uniqueness problem of a very wider class of equations.

On the other hand, it has been shown in [2] that the Lyapunov function is applicable to the uniqueness problem for differential equations and also shown in [1] that some estimations for solutions of differential inequalities yield the uniqueness theorem for differential equations.

In this paper, a functional-differential inequality including an operator T such that

$$|x'-f(t, x, Tx)|\leqq\varepsilon(t),$$

in which a functional-differential equation corresponds to the case $\varepsilon(t)\equiv 0$, will be considered as well as the existence problem for

$$(2) \qquad x'=f(t, x, Tx).$$

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