# RELATIONS BETWEEN DOMAINS OF HOLOMORPHY AND MULTIPLE COUSIN'S PROBLEMS

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## Introduction.

Oka [12] proved that a domain $D$ of holomorphy in $C^n$ is a *Cousin*-I *domain*, that is, any additive Cousin's distribution in $D$ has a solution. On the other hand from Cartan [5]-Behnke-Stein [2]'s theorem, a Cousin-I domain in $C^2$ is a domain of holomorphy. In this way any domain of holomorphy in $C^2$ can be completely characterized by additive Cousin's problems. For $n \geqq 3$, however, Cartan [6] showed that a Cousin-I domain in $C^n$ is not necessarily a domain of holomorphy. In the previous paper [10] we tried to characterize a domain of holomorphy in a Stein manifold by additive Cousin's problems. An open set $G$ in $C^n$ is called *regular* if $G \cap P$ is a Cousin-I open set for any relatively compact polycylinder $P$ in $C^n$. We proved that a domain in $C^n$ is a domain of holomorphy if and only if it can be exhausted by regular domains. Moreover, we proved that a regular open set is pseudoconvex in the Cartan's sense at its continuous boundary point. Making use of the results of Oka [13] or Docquier-Grauert [7] respectively, we proved that a domain in $C^n$ or more generally in a Stein manifold with a smooth boundary is a domain of holomorphy if and only if it is locally regular at its each boundary point.

Concerning multiple Cousin's problems the situation is more or less different. Thullen [16] gave an example of a domain in $C^2$ which is not a domain of holomorphy but a *Cousin*-II *domain*, that is, a domain in which any multiple Cousin's distribution has a solution. Let $\mathfrak{O}$ and $\mathfrak{O}^*$ be, respectively, the sheaves of all germs of holomorphic mappings in $C$ and $GL(1, C)$. As we remarked in [9], Thullen's example is a Cousin-II domain $D$ with $H^1(D, \mathfrak{O}^*) \neq 0$. In the previous paper [11] we proved that a domain $(D, \varphi)$ over $C^n$ with $H^1(D, \mathfrak{O}^*) = H^1(\varphi^{-1}(H), \mathfrak{O}^*) = 0$ for any analytic plane $H$ in $C^n$ is a domain of holomorphy. Especially a domain $(D, \varphi)$ over $C^2$ satisfies $H^1(D, \mathfrak{O}^*) = 0$ if and only if $(D, \varphi)$ is a domain of holomorphy with $H^2(D, Z) = 0$ where $Z$ is the abelian group of all integers. These facts suggest that we should obtain a sufficient condition that a domain $D$ in $C^n$ is a domain of holomorphy, if we put a similar discussion forward as in [10] substituting a domain $G$ with $H^1(G, \mathfrak{O}^*) = 0$ in stead of a Cousin-I domain.

As a polycylinder $P$ does not necessarily satisfy $H^1(P, \mathfrak{O}^*) = 0$, we shall consider only simply connected polycylinders in the definition below. An open set $G$ in $C^n$ is called *regular** if $H^1(G \cap P, \mathfrak{O}^*) = 0$ for any relatively compact and simply connected polycylinder $P$ in $C^n$. In the present paper we shall prove that a domain

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