ON A CHARACTERIZATION OF REGULARLY BRANCHED THREE-SHEETED COVERING RIEMANN SURFACES

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§ **1.** Let R be an open Riemann surface. Let $\mathfrak{M}(R)$ be a family of non-constant meromorphic functions on R. Let f be a member of $\mathfrak{M}(R)$. Let $P(f)$ be the number of Picard's exceptional values of f, where we say α a Picard's exceptional value of f when α is not taken by f on R. Let $P(R)$ be a quantity defined by

$$\sup_{f\in\mathfrak{M}(R)} P(f).$$

When R is open, we have always $P(R)\geqq 2$, since there exists a non-constant regular function on R by the existence theorem due to Behnke-Stein and then it suffices to compose it to the exponential function.

Ozawa [2] gave the following criterion of non-existence of analytic mapping between two Riemann surfaces:

If $P(R)<P(S)$, then there is no analytic mapping from R into S.

In general it is very difficult to calculate $P(R)$ of a given open Riemann surface.

Let R be an ultrahyperelliptic surface, which is a proper existence domain of a two-valued function $\sqrt{g(z)}$ with an entire function $g(z)$ of z whose zeros are all simple and are infinite in number. Then by Selberg's generalization [5] of Nevanlinna's theory we have $P(R)\leqq 4$. Ozawa [2, 3] gave a characterization of R with $P(R)=4$, an example of R with $P(R)=3$ and several other interesting results.

We shall confine ourselves to the following Riemann surfaces:

Let R be a regularly branched three-sheeted covering Riemann surface, which is a proper existence domain of the three-valued algebroid function $\sqrt[3]{g(z)}$ with an entire function $g(z)$ of z whose zeros are all simple or double and are infinite in number. Then by Selberg's theory [5] we have $P(R)\leqq 6$. The existence of the surface with $P(R)=6$ is evident.

In the present paper we shall prove the following theorems:

THEOREM 1. *If $P(R)=6$, then there exist entire functions $f(z)$, $H(z)$ of z such that*

$$(1.1)\qquad f(z)^3 g(z)=(e^{H(z)}-\gamma)(e^{H(z)}-\delta)^2,\qquad \gamma\neq\delta,\quad \gamma\delta\neq 0,$$

where $H(z)$ is a non-constant function with $H(0)=0$ and γ and δ are constants. The converse is also true.

THEOREM 2. *There is no regularly branched three-sheeted covering Riemann*

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