ON MATCHING METHODS IN TURNING POINT PROBLEMS

BY TOSHIHIKO NISHIMOTO

§ 1. Introduction.

We consider here the asymptotic nature of solutions of linear differential equations of the form

$$\varepsilon^h \frac{dy}{dx} = A(x, \varepsilon) y \tag{1.1}$$

as the parameter ε tends to zero. Here $A(x, \varepsilon)$ is 2-by-2 matrix such that

$$\begin{pmatrix} 0 & 1 \\ x^q + \varepsilon\phi(x, \varepsilon) & 0 \end{pmatrix}, \tag{1.2}$$

where the function $\phi(x, \varepsilon)$ is holomorphic in the complex variables x and ε in a domain of the x, ε-space defined by the inequalities

$$|x| \leqq x_0 < 1, \qquad 0 < |\varepsilon| \leqq \varepsilon_0, \qquad |\arg \varepsilon| \leqq \delta_0 \tag{1.3}$$

and $\varepsilon\phi(x, \varepsilon)$ has a uniformly asymptotic expansion in powers of ε such that

$$\varepsilon\phi(x, \varepsilon) \simeq \sum_{\nu=1}^{\infty} \phi_\nu(x) \varepsilon^\nu \tag{1.4}$$

as ε tends to zero in the domain (1. 3) with the coefficients $\phi_\nu(x)$ holomorphic for $|x| \leqq x_0$:

$$\phi_\nu(x) = \sum_{\mu=m_\nu}^{\infty} \phi_{\nu\mu} x^\mu, \qquad \phi_{\nu, m_\nu} \neq 0, \tag{1.5}$$

where $m_\nu \geqq 0$.

The quantities h and q are positive integers. Thus the equation (1. 1) has a turning point at the origin. When $h=1$ and $q=1$ or $h=1$ and $q=2$, the asymptotic solutions of the equation (1. 1) were constructed by Langer [3] and Mckelvey [4]. Their methods are the reductions of the given equations to simpler related problems which can be solved by explicit technique. And for $h=1$ and any positive integers q, Sibuya [2] found some simpler related equations, but the analyses of them are seen not to be completed. On the other hand, Wasow [6], [7] claimed that the matching methods are also fruitful in fairly general cases. He treated the system (1. 1) with $h=1$ and an n-by-n matrix $A(x, \varepsilon)$. The matrix $A(x, \varepsilon)$ has an asymptotic expansion

Received March 22, 1965.