ON COMPLEX ANALYTIC MAPPINGS BETWEEN TWO ULTRAHYPERELLIPTIC SURFACES

By Mitsuru Ozawa

§ **1.** Let R and S be two ultrahyperelliptic surfaces defined by two equations $y^2=G(z)$ and $u^2=g(w)$, respectively, where G and g are two entire functions each of which has no zero other than an infinite number of simple zeros. Let φ be an analytic mapping from R into S. Let $\mathfrak{P}_S$ be the projection map $(w, u)\to w$. Let Φ be the sifted mapping $\mathfrak{P}_S\circ\varphi$, then Φ is an entire function on R. Let $T(r, \Phi)$ be the Nevanlinna-Selberg characteristic function of Φ. Let $N(r\ R)$ be the quantity $N(r, \mathfrak{X})$ defined by Selberg [8], which is essentially one half of the integrated Euler characteristic of R defined by Sario [5].

Definition 1. *If $T(r, \Phi)$ satisfies the inequality*

$$\varlimsup_{r\to\infty}\frac{N(r, R)}{T(r, \Phi)}>2,$$

then we say φ a semi-degenerate analytic mapping from R into S.

Let $\mathfrak{P}_R$ be the projection map $(z, y)\to z$. If φ satisfies $\mathfrak{P}_S\circ\varphi(p)=\mathfrak{P}_S\circ\varphi(q)$ for $p\neq q$, $\mathfrak{P}_R p=\mathfrak{P}_R q$, then we say that φ satisfies the rigidity of projection map.

Definition 2. *If φ satisfies the rigidity of projection map, then we say φ a rigid analytic mapping from R into S.*

In the present paper we shall prove the following somewhat interesting

Theorem 1. *If φ exists and is a rigid analytic mapping from R into S, then there exists a suitable entire function $h(z)$ of z in such a manner that $f(z)^2G(z)=g\circ h(z)$ for a suitable entire function $f(z)$ of z.*

If φ is a semi-degenerate analytic mapping from R into S, then it is a rigid analytic mapping. If Φ is not single-valued with respect to z, we have $N(r, R)<2T(r, \Phi)+O(1)$ by Selberg's ramification theorem and hence

$$\varlimsup_{r\to\infty}\frac{N(r, R)}{T(r, \Phi)}\leqq 2$$

holds in our ultrahyperelliptic case. This contradicts the semi-degeneracy. Thus Φ must be single-valued for z, which is the desired rigidity of φ.

Received February 11, 1965.