A NOTE ON ALGEBRAS OF REAL-ANALYTIC FUNCTIONS

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1. Let M and N be real-analytic manifolds with countable topology. We consider the totality of (real-valued) real-analytic functions on M and N; these form in a natural way function algebras $C(M)$ and $C(N)$, respectively. In this note we give a simple proof of the following theorem:

THEOREM 1. *Assume that we have an isomorphism $\tilde{\phi}$ of $C(M)$ onto $C(N)$. Then there exists a real-analytic homeomorphism Φ of N onto M such that $\tilde{\phi}(f)(y) = f(\Phi(y))$ $(f \in C(M))$.*

Also we try to give a characterization of maximal ideal of $C(M)$.

It is well known that a result analogous to the above theorem holds in differentiable case [1]. Its proof, however, cannot be directly applicable to the above theorem since the method of localization through the partition of unity becomes infeasible in real-analytic case. Alternatively, use is made here of the following Cartan's result [2].

PROPOSITION 1. *Let $f_i(x)$ be a finite number of real-analytic functions on M such that $f_i(x)$ have no common zero-points. Then there exist real-analytic functions $\varphi_i(x)$ on M satisfying $\sum f_i \varphi_i \equiv 1$.*

Now, let ζ be a homomorphism of $C(M)$ onto the real number field $\boldsymbol{R}$. We call such ζ a character of $C(M)$. Put $A_\zeta = \zeta^{-1}(0)$. It is clear that A_ζ is a maximal ideal of $C(M)$.

LEMMA 1. *For a given character ζ, there exist a compact set K and a real-analytic function f of A_ζ such that f has no zero-point outside of K.*

Proof. There is a real-analytic function g on M with the property that for any real number α the set $S_\alpha = \{x \mid g(x) = \alpha\}$ is a compact set. Actually, the existence of such a g is an immediate consequence of the fact that M can be properly imbedded in a Euclidean space via real-analytic map such that its image becomes a closed set [3]. Then $f = g - \zeta(g) \cdot 1$ is in A_ζ, while the zero-points of f form a compact set. Hence f is a desired function.

LEMMA 2. *Any A_ζ is given by a maximal ideal of $C(M)$ which consists of the functions vanishing at a single point $x_\zeta \in M$.*

Proof. Let f be a real analytic function of A given in Lemma 1. The zero-points of f are contained in a compact set K. We show that real-analytic functions

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