RICCI'S FORMULA FOR NORMAL GENERAL CONNECTIONS AND ITS APPLICATIONS

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In [15], Ōtsuki gave a kind of Ricci's formulas for a space with an integrable normal general connection, that is the distribution of the tangent subspaces of the space associated with the connection is completely integrable. In the present paper, the authors will give a generalized Ricci's formula without the condition of integrability and its applications for induced general connections on subspaces.

§1. Preliminaries.

Let $\mathfrak{X}$ be an n-dimensional differentiable manifold with a general connection[1] which is written in terms of local coordinates u^i as

$$\gamma=\partial u_j\otimes(P_i^j d^2u^i+\Gamma_{ih}^j du^i\otimes du^h), \tag{1.1}$$

where $\partial u_j=\partial/\partial u^j$ and d^2u^i denotes the differential of order 2 of u^i.

The components of the curvature tensor of γ are given by

$$R_i{}^j{}_{hk}=\left\{P_l^j\left(\frac{\partial\Gamma_{mk}^l}{\partial u^h}-\frac{\partial\Gamma_{mh}^l}{\partial u^k}\right)+\Gamma_l{}^j{}_h\Gamma_m{}^l{}_k-\Gamma_l{}^j{}_k\Gamma_{mh}^l\right\}P_i^m \tag{1.2}$$

$$-\delta_{m,h}^j\Lambda_{ik}^m+\delta_{m,k}^j\Lambda_{ih}^m,^{1)}$$

where

$$\Lambda_{ih}^j=\Gamma_{ih}^j-\frac{\partial P_i^j}{\partial u^h}$$

and $\delta_{m,h}^j$ denote the covariant derivarives of the Kronecker's δ_m^j with respect to γ.

γ is called *normal* when the tensor $P=P_i^j\partial u_j\otimes du^i$ of type (1, 1) is normal.[2] Let Q be the tensor such that $Q=P^{-1}$ on the image of P and $Q=P$ on the kernel of P at each point of $\mathfrak{X}$ regarding P as a homomorphism of the tangent bundle $T(\mathfrak{X})$ of $\mathfrak{X}$. The tensor field $A=PQ=QP$ with local components A_i^j is called the *canonical projection* of γ. The components $'R_i{}^j{}_{hk}$ and $''R_i{}^j{}_{hk}$ of the curvature tensors of the contravariant part $'\gamma=Q\gamma$ and the covariant part $''\gamma=\gamma Q$ of the normal general connection γ can be written respectively as

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1) See [8], §6.

2) See [11], §1.