ON AN ESTIMATE FOR SEMI-LINEAR ELLIPTIC DIFFERENTIAL EQUATIONS OF THE SECOND ORDER WITH DINI-CONTINUOUS COEFFICIENTS

By Yoshikazu Hirasawa

§1. Introduction.

In this paper, we are concerned with the a priori estimate for derivatives of solutions of the semi-linear elliptic differential equation

$$\sum_{i,j=1}^{n} a_{ij}(x) \frac{\partial^2 u}{\partial x_i \partial x_j} = f\left(x, u, \frac{\partial u}{\partial x}\right).^{1)} \tag{1. 1}$$

Concerning the estimate of this sort, Nagumo obtained a result on the assumption that the coefficients $a_{ij}(x)$ satisfy the Lipschitz condition ([4],[2] pp. 211–215, Theorem 2), and thereafter Simoda [5] and the author [3] improved Nagumo's result on the assumption that the coefficients $a_{ij}(x)$ satisfy the Hölder condition.

As, however, it is desirable from a theoretical point of view, that we have the a priori estimate under the weakest possible condition on the continuity of the coefficients $a_{ij}(x)$, we shall form, in this paper, an a priori estimate of the same type as obtained in the above-cited papers, provided that the coefficients $a_{ij}(x)$ satisfy the Dini condition.

The Dini condition which we impose on the coefficients $a_{ij}(x)$, is more restrictive than usual, but it seems to be considerably general.

Our method of proof in this paper is analoguous to one in the previous paper [3], which was composed of Nagumo's one and Cordes' modified results [2]. Therefore, the parts of the proof which can be carried out in the same way as in the previous paper, will often be omitted.

In § 2, we shall give two definitions concerning Dini functions, and prove two lemmas in regard to the properties of Dini functions given in these two definitions.

In § 3, we state the main result of this paper, whose proof is left to § 5. A set of lemmas will be made in § 4, and two other results will be proved in § 6.

§2. Dini functions $\varphi(t)$.

In this section, we make some preliminary remarks on Dini functions $\varphi(t)$ which define the modulus of continuity of the coefficients $a_{ij}(x)$ in the differential

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1) $\partial u/\partial x$ denotes the n-dimensional real vector $(\partial u/\partial x_1, \partial u/\partial x_2, \cdots, \partial u/\partial x_n)$.

2) The numbers in the brackets refer to the list of references at the end of this paper.