ON PARALLEL SLIT MAPPINGS

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Dedicated to Professor K. Kunugi on his sixtieth birthday

1. Let Ω be a plane domain containing ∞. In the family of all the univalent functions f with the expansion $f(z)=z+c/z+\cdots$ about ∞, there exists a function maximizing Re $e^{-2i\alpha}c$. It is known to be unique, which will be denoted by p^{α}. We are interested in the function

$$\varphi=p^{0}+p^{\pi/2},$$

whose importance is well known.

It has been proved that, if Ω is bounded by a finite number of analytic Jordan closed curves, then φ is univalent and, almost simultaneously, every boundary component of its image domain is a *convex analytic Jordan closed curve.* The proof is found in, e.g., Schiffer [5] or Ahlfors-Beurling [1]. People say that their proofs involve a mistake, but it is also universally admitted that a small technical modification makes the proof correct.

For an arbitrary Ω, by an approximation through the exhaustion of the domain, it is easily seen that φ is still univalent. The shape of the boundary of the image domain was discussed by Sario [4], who showed that a boundary component of $\varphi(\Omega)$ is *either a point, or a line segment or else a convex curve.* Unfortunately the latter half of his proof seems to be based on an incomplete discussion.

It is the purpose of the present note to give a complete proof to Sario's theorem.

2. We mean by a ***boundary component*** Γ of Ω a (connected) component of the boundary of Ω. It is equivalent to the one in the sense of Kerékjártó-Stoïlow. If f is univalent on Ω we see, by the latter definition, a boundary component of $f(\Omega)$ corresponds to Γ canonically, which we shall denote by $f(\Gamma)$. Let us remark that a point w belongs to $f(\Gamma)$ if and only if $w=\lim f(z_n)$ for a sequence of points $z_n\in\Omega$ having limit points only on Γ.

3. For an arbitrary Ω, every boundary component of $p^{\alpha}(\Omega)$ is known to be either a point or a line segment with inclination α. In general this property is not sufficient to characterize p^{α}, but so does if Ω is of finite connectivity. This shows that the following (1) holds for Ω of finite connectivity. The approximation through

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