CONFORMAL MAPPING ONTO POLYGONS BOUNDED BY SPIRAL ARCS

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0. Let G_0 be a simply-connected bounded domain laid on the complex w-plane which is starlike with respect to $w=0$. Suppose that the boundary of G_0 consists of a finite number of circular arcs centred at the origin and of rectilinear segments on rays issuing from the origin. Denote by $w=f(z)$ an analytic function which maps $|z|<1$ univalently onto G_0 and satisfies a normalization $f(0)=0$. Goodman [1] established an integral representation formula of Schwarz-Christoffel type for such a mapping function.

In a recent paper [5], one of the authors has given an alternative proof for Goodman's result. It may be noticed from the proof in [5] that the starlikeness of G_0 as well as its one-sheetedness are not essential. In fact, the whole procedure of this proof remains valid without any substantial modification. Further, as remarked below, it is also admissible that the domain G_0 has infinite protrusions and its boundary contains circular or rectilinear slits.

Now, both (infinitely winding) circumferences centred at the origin and rays issuing from the origin may be regarded as particular extreme members of a wider class of curves which consists of all logarithmic spirals with the asymptotic point at the origin; this is a fact which was ingeniously used by Grunsky [2]. In fact, any logarithmic spiral with the asymptotic point at the origin is expressed by the equation

$$\arg w - \lambda \lg w = c.$$

The parameter λ representing the inclination of the spiral runs over real numbers, while the parameter c may be restricted by $0 \leqq c < 2\pi$. More precisely, the value of χ determined by $\lambda = \tan(\chi/2)$ denotes the constant angle of the tangent at every point of the spiral to the radius vector through the point. A limiting case $\lambda=0$, $\chi=0$ corresponds to a ray starting at the origin, while another limiting case $\lambda=\infty$, $\chi=\pi$ ($\lg|w|=\text{const}$) corresponds to a circumference about the origin.

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