INFINITESIMAL TRANSFORMATIONS OF A MANIFOLD WITH f-STRUCTURE

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Professor Yano [3] introduced the concept of f-structure on an n-dimensional differentiable manifold and investigated it from the global viewpoint. The f-structure may be regarded as a generalization of the almost complex structure and the almost contact structure. The main purpose of this paper is to study such an infinitesimal transformation v^h of a differentiable manifold with f-structure as leaves the structure tensor f_i^h invariant, that is, $\underset{v}{\mathcal{L}} f_i^h=0$.

§ 1. Preliminaries.

We consider an n-dimensional differentiable manifold of class C^∞ covered by a system of coordinate neighborhoods $\{x^h\}$, and a tensor field f_i^h of type (1, 1) and of class C^∞ satisfying

$$f_i^t f_t^s f_s^h + f_i^h = 0, \tag{1. 1}$$

where the Latin indices run over 1, 2, ⋯, n.

In a manifold with (1. 1), the operations

$$l_i^h = -f_i^t f_t^h \qquad \text{and} \qquad m_i^h = f_i^t f_t^h + \delta_i^h \tag{1. 2}$$

applied to the tangent space at a point of the manifold are complementary projection operators. Thus there exist complementary distributions L and M corresponding to the projection operators l_i^h and m_i^h, respectively.

If the rank of f is r, then we call such a structure an *f-structure of rank r* ($r \leqq n$). If the rank of f is n, then $l_i^h = -\delta_i^h$ and $m_i^h = 0$, so that we find that the f-structure of rank n is an almost complex structure. And if the rank of f is $n-1$, then the distribution L is $(n-1)$-dimensional and the distribution M is one dimensional, consequently m_i^h should have the form $m_i^h = p^h q_i$, where p^h and q_i are contravariant and covariant vector fields respectively. Therefore, we find that the f-structure of rank $(n-1)$ is an almost contact structure defined by Sasaki [1]. (Yano [3].)

Making use of (1. 1) and (1. 2), we find

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