ON INFINITESIMAL TRANSFORMATIONS OF ALMOST-KÄHLERIAN SPACE AND K-SPACE

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1. Introduction.

In the present paper, we shall consider mainly infinitesimal conformal and projective transformations of almost-Kählerian space, K-space and, as their special case, Kählerian space. Under what conditions do these transformations become isometric? Even though there are many papers about this problem, it seems to the author that there exist few about non-compact spaces. Recently Couty [1] proved that in an almost-Kähler-Einstein space with positive scalar curvature, an infinitesimal projective transformation is necessarily an isometry and Tashiro [7] proved that in a Kählerian space with non-vanishing constant curvatue scalar, an infinitesimal conformal transformation is necessarily an isometry. In this paper, we shall deal with the same problem but throughout this paper we do not assume that the space is compact. In § 2 we shall state some properties of almost-Kählerian space and K-space for later use. In § 3 we shall obtain sufficient conditions for an infinitesimal conformal transformation to be an isometry and especially a condition corresponding to Couty's result on a projective transformation and give a decomposition of an infinitesimal conformal transformation in an Einstein K-space. In § 4 we shall deal with the same problem of an infinitesimal projective transformation. A remark on the result obtained by Tachibana [4] about an infinitesimal analytic conformal transformation in a K-space will be given in the last § 5.

2. Almost-Kählerian space and K-space.

Let X_{2n} be a $2n$-dim. almost-complex space[1] and $\varphi_j{}^i$ its almost-complex structure, then by definition we have

$$\varphi_j{}^s\varphi_s{}^i=-\delta_j{}^i. \tag{2.1}$$

An almost-complex space with a positive definite Riemannian metric g_{ji} satisfying

$$g_{rs}\varphi_j{}^r\varphi_i{}^s=g_{ji} \tag{2.2}$$

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1) For example, see Yano [8]. Indices run over 1, 2, ···, $2n$.