REMARKS ON UNRAMIFIED ABELIAN COVERING SURFACES OF A CLOSED RIEMANN SURFACE

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1. Introduction. Let R be a closed Riemann surface of genus p and W be an unramified unbounded regular covering surface of R whose covering transformation group $\Gamma(W)$ is abelian. Let C_{2i-1}, C_{2i} $(i=1, \cdots, p)$ be $2p$ canonical homology basis of R. Then $\Gamma(W)$ may be considered as an abelian group generated by C_i $(i=1, \cdots, 2p)$ with a number of defining relations among them:

$$\sum_{i=1}^{2p} r_{ki} C_i = 0, \qquad k=1, \cdots, q \ (0 \leqq q \leqq 2p)$$

with integral coefficients r_{ki}, whose $q \times 2p$ matrix

$$(r_{ki})$$

is of rank q. The rank r of $\Gamma(W)$ is defined by $2p-q$.

Mori [1] proved the following theorem for this surface W:

(1). $W \in O_G$ if and only if $r \leqq 2$.

(2). $W \in O_{AD}$.

(3). $W \in O_{AB}$ if there exists, for each $i=1, \cdots, p$, a relation of the form

$$r_{2i-1} C_{2i-1} + r_{2i} C_{2i} = 0$$

with not both vanishing integral coefficients r_{2i-1} and r_{2i}. Especially this is the case, when W consists of a (finite or infinite) number of replicas of a planar surface obtained from R by cutting along p disjoint non-dividing loop cuts.

Let O_{MD} denote the class of Riemann surfaces not tolerating non-constant single-valued analytic function with a finite spherical area. Let O_{AB}^0 denote the class of Riemann surfaces any subregion of which tolerates no non-constant bounded analytic function whose real part vanishes continuously on its relative boundary.

In the present paper we shall prove the following theorem:

THEOREM 1. *Let W be an unramified unbounded regular abelian covering surface of R. If W satisfies the condition in* (3) *and $r \geqq 3$, then*

$$W \in O_{AB}^0 \frown O_{MD}.$$

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