ON THE GROWTH OF ANALYTIC FUNCTIONS

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1. In our previous papers [3], [4] we made use of the rigidity of projection map in order to establish some results on the value distribution of analytic functions on some Riemann surfaces. In the present paper we shall construct two open Riemann surfaces on which there is no analytic function of order lower than any given number. In the first example our Riemann surface belongs to the class O_G. In the second example we shall construct an open Riemann surface of hyperbolic type having the similar property. Our fundamental tools are (1) the rigidity property of projection map for functions of lower growth and (2) the unsymmetric welding of two surfaces.

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2. The first example. Let E be a plane with an infinite number of slits S_j clustering only at the point at infinity. We assume that S_1 is the unit segment $[0, 1]$. Let E_1 and E_2 be two copies of E. We shall connect E_1 and E_2 along each slits S_j $(j>1)$ in the standard manner and along S_1 in the following manner. Let $\sigma(t)$ be a monotone increasing function in $[0, 1]$ such that

$$\sigma(t)=t+t^2(t-a)^2(t-b)^2(t-1)^2, \qquad 0<a<b<1.$$

The upper shore S_1^+ of S_1 on E_1 and the lower shore S_1^- of S_1 on E_2 are welded in such a manner that $t \in S_1^+$ corresponds to $\sigma(t) \in S_1^-$. The lower shore of S_1 on E_1 and the upper shore of S_1 on E_2 are welded at the points with the same coordinate. This process is called a σ-process. There are many other σ-processes. The resulting surface is denoted by W, which is a Riemann surface belonging to the class O_G and two points 0 and 1 correspond to two inner points of new surface W. See Courant [1], p. 69 "Sewing theorem".

Let $n(r, E)$ be the number of end points of slits S_j $(j>1)$ lying in $|z|<r$. Let $T(r, f)$ be the Nevanlinna-Selberg characteristic of f on W over the ring domain $r_0<|z|<r$, $r_0>2$ or more general one defined by Sario.

THEOREM. *There is no non-constant single-valued meromorphic function f on W satisfying the growth condition*

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