CERTAIN ALMOST CONTACT HYPERSURFACES IN EUCLIDEAN SPACES

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Introduction.

An odd-dimensional differentiable manifold M^{2n+1} is said to have an almost contact structure or to be an almost contact manifold if the structural group of its tangent bundle is reducible to the product of a unitary group with the 1-dimensional identity group [3].[1] Recently Sasaki and Hatakeyama [4, 5] proved that an almost contact structure is equivalent to the existence of a set of tensor fields ϕ, ξ, η of the type (1, 1), (1, 0) and (0, 1) satisfying the following five conditions:

$$\xi^i\eta_i = 1, \tag{0. 1}$$

$$\text{rank}\,(\phi_j{}^i) = 2n, \tag{0. 2}$$

$$\phi_j{}^i\xi^j = 0, \tag{0. 3}$$

$$\phi_j{}^i\eta_i = 0, \tag{0. 4}$$

$$\phi_j{}^i\phi_k{}^j = -\delta_k^i+\eta_k\xi^i. \tag{0. 5}$$

This permits us to study almost contact structures by use of the tensor calculus. They also proved that we can introduce an associated Riemannian metric tensor which satisfies both of the relations

$$g_{ji}\xi^i = \eta_j, \tag{0. 6}$$

$$g_{rs}\phi_i{}^r\phi_j{}^s = g_{ij} - \eta_i\eta_j. \tag{0. 7}$$

We call an almost contact metric structure an almost contact structure with this associated Riemannian metric.

On the other hand, Tashiro [7] proved that any orientable differentiable hypersurface in an almost Hermitian manifold admits an almost contact structure and that the Riemannian metric induced on the hypersurface is an associated metric of the almost contact structure.

Thus, an even-dimensional Euclidean space E^{2n} being regarded as a flat Kaehlerian manifold, any differentiable hypersurface of E^{2n} has an induced almost contact metric structure. The purpose of this paper is to study certain almost contact hypersurfaces and to show that, in a Euclidean space E^{2n}, only E^{2n-1}, S^{2n-1} and $E^r\times S^{2n-r-1}$ can admit induced normal (see [6]) almost contact metric structure.

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1) The numbers in the brackets refer to the bibliography at the end of the paper.