RIGIDITY OF PROJECTION MAP AND THE GROWTH OF ANALYTIC FUNCTIONS

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1. In our previous paper [3] we determined the exceptional class of Picard's theorem on some Riemann surfaces with conformal automorphisms by the aid of the existence of the fundamental functions and the rigidity of projection map which was made use of in a non-emphasized form. We shall here explain it explicitly in the following form. (We make use of the same notations as in [3].)

Let W belong to $\mathfrak{G}_2 \frown P_{MD}$. Let $f(p)$ be a single-valued meromorphic function on an end Ω of W satisfying $T(\sigma)=o(e^{2\sigma})$, then we have the representation

$$f(p) = F \circ F_0(p)$$

for a single-valued meromorphic function $F(w)$ in the punctured disc $\sigma_0 < \log |w| < \infty$ satisfying the condition $T_P(\sigma, F)=o(e^{2\sigma})$.

This fact says that $f(p)$ preserves the projection map F_0: $W \rightarrow \mathfrak{W}^*$, that is, $f(p_1)=f(p_2)$ if $F_0(p_1)=F_0(p_2)$, when $f(p)$ satisfies the desired growth condition. Such a phenomenon was studied non-systematically by the various authors. Excepting the closed surface case, the first one who explained the phenomenon is Selberg [5]. However his ramification theorem in his celebrated theory [4], that is,

$$N(r; \mathfrak{X})<(2k-2)T(r; f)+O(1),$$

can be considered as a representation of the rigidity of projection map in terms of the growth condition. Myrberg [2] constructed a striking example which shows that the analytic theory meets a deep difficulty in the first step. In [1] Heins constructed an interesting example by using the phenomenon and proved an elegant composition theorem, which can be considered as an analytic representation of the rigidity of projection map by any bounded analytic function on an end. Heins' proof of the theorem was based upon the Schur algorithm. In [3] we made use of the Heins composition theorem in order to prove the existence of the fundamental functions.

In the present paper we shall give another proof of Heins' composition theorem. Further we shall give some examples, which show how the rigidity of projection map is important and effective.

2. *Heins' composition theorem*: Let W be an open Riemann surface of class O_G with the only one ideal boundary component. Let $f(p)$ be a non-constant bounded

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