# A FUNCTIONAL METHOD FOR STATIONARY CHANNELS

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*Dedicated to Professor K. Kunugi on his sixtieth birthday*

## 1. Introduction.

The concept of the finite memory channel of Shannon has been formulated in the purely mathematical form by McMillan and Khinchin (Cf. [10]), and established in the present elegant style by Feinstein [6]. Around the finite memory channels as its focal point, there exist various theorems, in which one of the most important results is the theorem of the equality of $C_s$ and $C_e$ of stationary and ergodic capacities. The problem 'whether the equality holds' has been an open question since Khinchin's paper [10]. This equality has been recently proved by many authors: Tsaregradsky [13], Carleson [3], Feinstein [7], Breiman [2], Parthasarathy [11] and others. In this paper we shall describe it in an abstract form.

The purpose of this paper is to introduce an abstract characterization of finite or infinite memory channel in which the input space and the output space are compact (totally disconnected) Hausdorff spaces with a pair of fixed homeomorphisms, and in which the channel distribution has a continuous property. In particular, every memory channel has always these properties. The usual memory channels are based upon their message symbols with practical applications. However their symbols may sometimes produce certain troublesome complications for the developments of several mathematical computations of them. The message symbols, in the present construction of the channel, will not be presented, and they will be replaced by sets with the property of the closed-openness (clopen, say). The descriptions will be given only by topological and functional forms, that is, they will be described by topological and Banach spaces methods. The entropy functional $H(\cdot)$ (cf. Umegaki [15] and [16]) and the transmission functional $\mathfrak{R}(\cdot)$ are defined over the Banach space of bounded signed regular measures, and they depend upon a clopen partition, or upon a pair of such partitions in the input and output spaces.

In §2, in order to clarify the abstract stationary channel $(X, \nu, Y)$ defined below, the definition of stationary finite memory channel $(A^I, \nu, B^I)$ will be first stated with respect to the conditions (m 1)$\sim$(m 5). These conditions will be replaced below by the conditions (C 1)$\sim$(C 5) in the channel $(X, \nu, Y)$, respectively. In §3, several notations and preliminaries will be given, and in §4 the stationary channel $(X, \nu, Y)$

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