ON FUBINIAN AND *C*-FUBINIAN MANIFOLDS

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In his previous papers[1], one of the present authors proved that an orientable hypersurface in an almost complex manifold has an almost contact structure and obtained a condition[2] in order that a hypersurface in a Kählerian manifold is Sasakian. In the present paper, a hypersurface satisfying the condition will be called a *C*-umbilical hypersurface. A manifold having the same Sasakian structure as a *C*-umbilical hypersurface in a locally Fubinian manifold will be said to be locally *C*-Fubinian. The purpose of the present paper is to show some characteristic properties of Fubinian and *C*-Fubinian manifolds.

§ 1. Preliminaries.

Let M be a $2n$-dimensional almost Hermitian manifold with almost complex structure $F=(F_\lambda{}^\kappa)$ and metric tensor $G=(G_{\mu\lambda})$. We shall denote the curvature tensor by $K_{\nu\mu\lambda}{}^\kappa$, the Ricci tensor by $K_{\mu\lambda}$, the scalar curvature by $\kappa=K_{\mu\lambda}G^{\mu\lambda}/2n(2n-1)$, and the covariant differentiation with respect to the Riemannian connection of the metric G by ∇_μ.

If M is Kählerian, we know the identities

$$K_{\nu\mu\lambda\kappa}F_\pi{}^\lambda F_\omega{}^\kappa=K_{\nu\mu\pi\omega}, \tag{1.1}$$

$$F^{\nu\mu}K_{\nu\mu\lambda\kappa}=-2K_\lambda{}^\omega F_{\omega\kappa}=2K_\kappa{}^\omega F_{\omega\lambda}. \tag{1.2}$$

For a vector $V=(V^\kappa)$, we put $\|V\|^2=G_{\mu\lambda}V^\mu V^\lambda$, $\tilde{V}^\kappa=-V^\lambda F_\lambda{}^\kappa$ and

$$K(V)=-K_{\nu\mu\lambda\kappa}\tilde{V}^\nu V^\mu\tilde{V}^\lambda V^\kappa/\|V\|^4, \tag{1.3}$$

$$R(V)=K_{\mu\lambda}V^\mu V^\lambda/\|V\|^2. \tag{1.4}$$

These quantities $K(V)$ and $R(V)$ are the so-called holomorphic sectional curvature and the Ricci curvature (belonging to the direction) of the vector V, respectively.

On the other hand, let $\bar{M}$ be a $(2n-1)$-dimensional almost Grayan manifold with structure (f, g) consisting of an almost contact structure

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1) Tashiro [5]. Terminologies and notations of the papers will be taken over in the present paper. The various kinds of indices run on the following ranges respectively:

$$\kappa, \lambda, \mu, \nu, \omega=1, \ldots, 2n;$$
$$h, i, j, k, l=1, \ldots, 2n-1;$$
$$A, B, C=1, \ldots, 2n-1, \infty.$$

2) See Theorem 8 in [5], or the equation (1.12) in the below.