SOME EXPANSION THEOREMS FOR STOCHASTIC PROCESSES, I

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1. Let $\mathcal{E}(t)$ $(-\infty<t<\infty)$ be a continuous stationary stochastic process of the second order (in the wide sense) with mean zero; that is,

$$E\{\mathcal{E}(t+u)\overline{\mathcal{E}(t)}\}=\rho(u) \tag{1. 1}$$

is a continuous function of u only, and

$$E\{\mathcal{E}(t)\}=0,\ -\infty<t<\infty. \tag{1. 2}$$

$\rho(u)$ is called the correlation function of $\mathcal{E}(t)$. We have, then,

$$\mathcal{E}(t)=\int_{-\infty}^{\infty} e^{it\lambda}\, dZ(\lambda) \tag{1. 3}$$

and

$$\rho(u)=\int_{-\infty}^{\infty} e^{iu\lambda}\, dF(\lambda), \tag{1. 4}$$

where $F(\lambda)$ is a bounded non-decreasing function such that $F(\infty)-F(-\infty)=\rho(0)=E\{|\mathcal{E}(t)|^2\}$, and $Z(\lambda)$ is an orthogonal process such that $E\{|Z(\lambda')-Z(\lambda)|^2\}=F(\lambda'-0)-F(\lambda-0)$. $F(\lambda)$ and $Z(\lambda)$ are called the spectral function and the random spectral function of $\mathcal{E}(t)$ respectively.

Let

$$X(t)=f(t)+\mathcal{E}(t),\ -\infty<t<\infty, \tag{1. 5}$$

and consider

$$n\int_{-\infty}^{\infty} X(t-s)\, K(ns)\, ds=\int_{-\infty}^{\infty} X\left(t-\frac{s}{n}\right)K(s)\, ds, \tag{1. 6}$$

where $f(t)$ and $K(s)$ are numerical valued functions. Kawata [5] has shown that if (i) $f(s)/(1+|s|^{3/2})\in L^1(-\infty, \infty)$, (ii) $f(t+u)-f(t)=O(u)$ for small u, (iii) $(1+|s|)\, K(s)$

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