ON THE ENVELOPE OF HOLOMORPHY OF A GENERALIZED TUBE IN C^n

BY JOJI KAJIWARA

In 1937 Stein [10] proved that the envelope of holomorphy of a tube-domain in C^2 coincides with its envelope of convexity. We can find no difficulty in extending the above Stein's proof to the case in C^n. In 1938 Bochner [2] obtained the above Stein's Theorem quite independently in C^n. Later Hitotumatu [7] gave a new and elegant proof and Bremermann [5] extended the above Stein's Theorem in complex Banach spaces.

The main purpose of the present paper is to extend the above Stein's Theorem to a generalized tube in C^n. The main method is based on the Levi's problem and the convergence theorem concerning the domain of holomorphy.

For two n-tuples $x=(x_1, x_2,\cdots, x_n)$ and $y=(y_1, y_2,\cdots, y_n)$ of real numbers, we shall use the notation $z=x+iy$ by putting $z=(z_1, z_2,\cdots, z_n)$ and $z_j=x_j+iy_j$ $(1\leqq j\leqq n)$. The space of n real variables $x_1, x_2,\cdots$ and x_n is denoted by R_x^n and the space of n complex variables $z_1, z_2,\cdots$ and z_n is denoted by C_z^n or simply by C^n.

Let A and B be subsets of R_x^n and R_y^n respectively. Then $A\times B$ is called a *generalized tube* in C_z^n where $z=x+iy$. A is called its *real base* and B is called its *imaginary base*. $A\times R_y^n$ is called simply a *tube* in C_z^n.

Concerning a tube in C^n we have the following theorem [10].

STEIN'S THEOREM. *The envelope of holomorphy of an open connected tube in C^n coincides with its geometrical envelope of convexity.*

LEMMA 1. *If an open connected generalized tube $A\times\{(y_1, y_2,\cdots, y_n); a_j<y_j<b_j$ $(j=1,2,\cdots,n)\}$ is a domain of holomorphy, then $A\times\{(y_1, y_2,\cdots, y_n); a_j+c_j<y_j<b_j+c_j$ $(j=1,2,\cdots,n)\}$ is also a domain of holomorphy for any real number c_j.*

Proof. Since the holomorphic mapping Φ defined by $\Phi(z)=(z_1+ic_1, z_2+ic_2,\cdots, z_n+ic_n)$ is a bi-holomorphic mapping of the closure of the former onto that of the latter, we have our Lemma. q. e. d.

LEMMA 2. *If an open connected generalized tube $T=A\times\{(y_1, y_2,\cdots, y_n); a_j<y_j<b_j$ $(j=1,2,\cdots,n)\}$ is not a domain of holomorphy, then for any positive integer k $(1\leqq k\leqq n)$ and for any real number d_k such that $(a_k+b_k)/2<d_k\leqq b_k$, $T_1=T\cap[A$*

Received December 7, 1962.